SCIENTIFIC CASE:

Gravitation: The mass of Jupiter[1]

Team members:

Writer: ____

Equipment manager: _________________________________

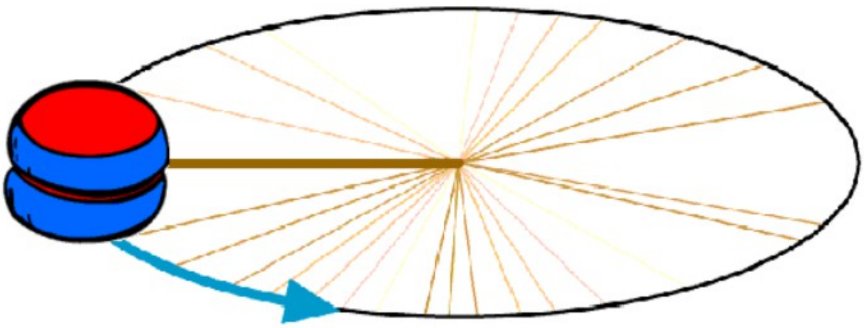
Reader: ____

Spokesperson: ____

Maths: ____

Context

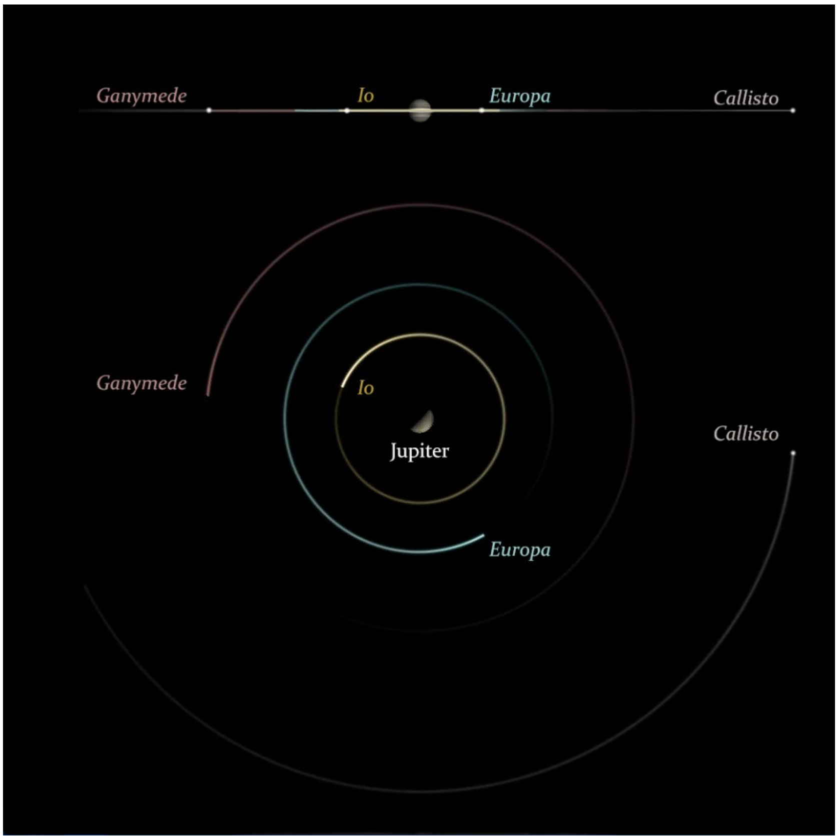
When a yo-yo rotates horizontally, it describes the following trajectory.:



1 *Educational material manufacturated by [“Asociación Planeta Ciencias”] under the initiative and coordination of the [European Space Agency] inside the [CESAR] program framework.*

That is, the yo-yo moves with uniform circular motion. (UCM)[2].

Similarly, we can study the planets' motion around the Sun, or that of a moon around its planet[3].



Orbits described by Jupiter's moons. Credit: CESAR

More educational resources:

CESAR: http://cesar.esa.int/index.php?ChangeLang=en
ESA education: http://www.esa.int/Education
CESAR Booklets: http://cesar.esa.int/index.php?Section=Booklets&ChangeLang=en

Extra activity:
http://cesar.esa.int/index.php?Section=The_mass_of_jupiter&ChangeLang=en

2 In order to go a bit further, we suggest you try a kinematics experiment in class, to make sure the theory complies with your observations. You'll find it at the end of this document.

3 Even though orbital motion is elliptical, circular motion is a good approximation for our case.

Scientific case: finding the mass of Jupiter

Galileo Galilei (1564 - 1642) discovered four of Jupiter's moons in 1610, which are commonly referred to as Galilean (or Jovian) moons (or satellites): **Io, Europa, Ganymede, and Callisto**. They are the biggest of this planets' satellites (they can even be seen with binoculars).

Using observational data obtained by Galileo's telescope, our aim is to **study their motion** and **find out the mass of Jupiter**.

- Galileo's observations dramatically contributed to the first great scientific revolution, and the downfall of the Ptolemaic Model (according to which the Earth is the center of the universe), to be replaced by the heliocentric model (the Sun is the center). -

Research material

You have access to the following:

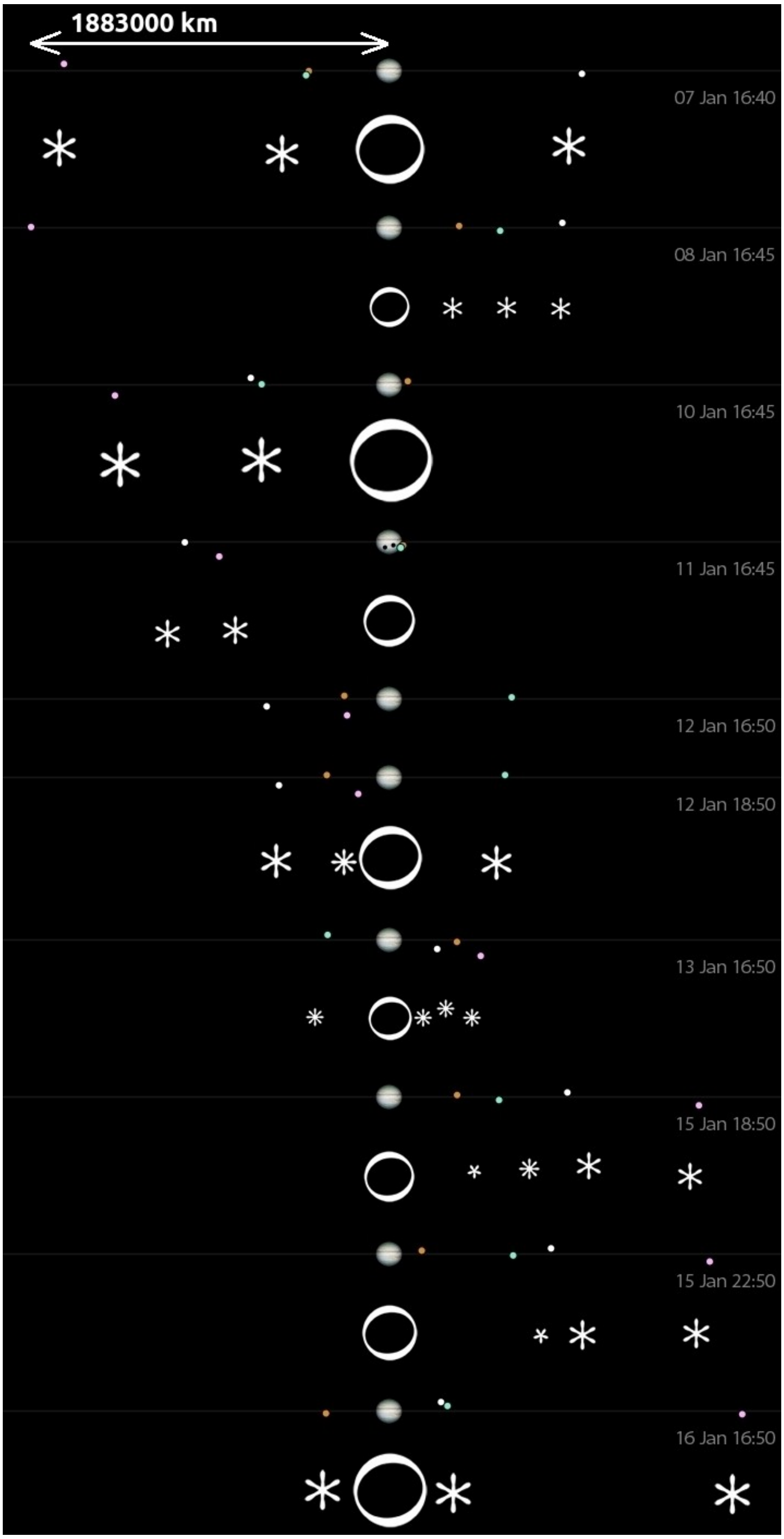
- Pencils, paper, rubber, ruler.
- Galileo's first satellite observations.

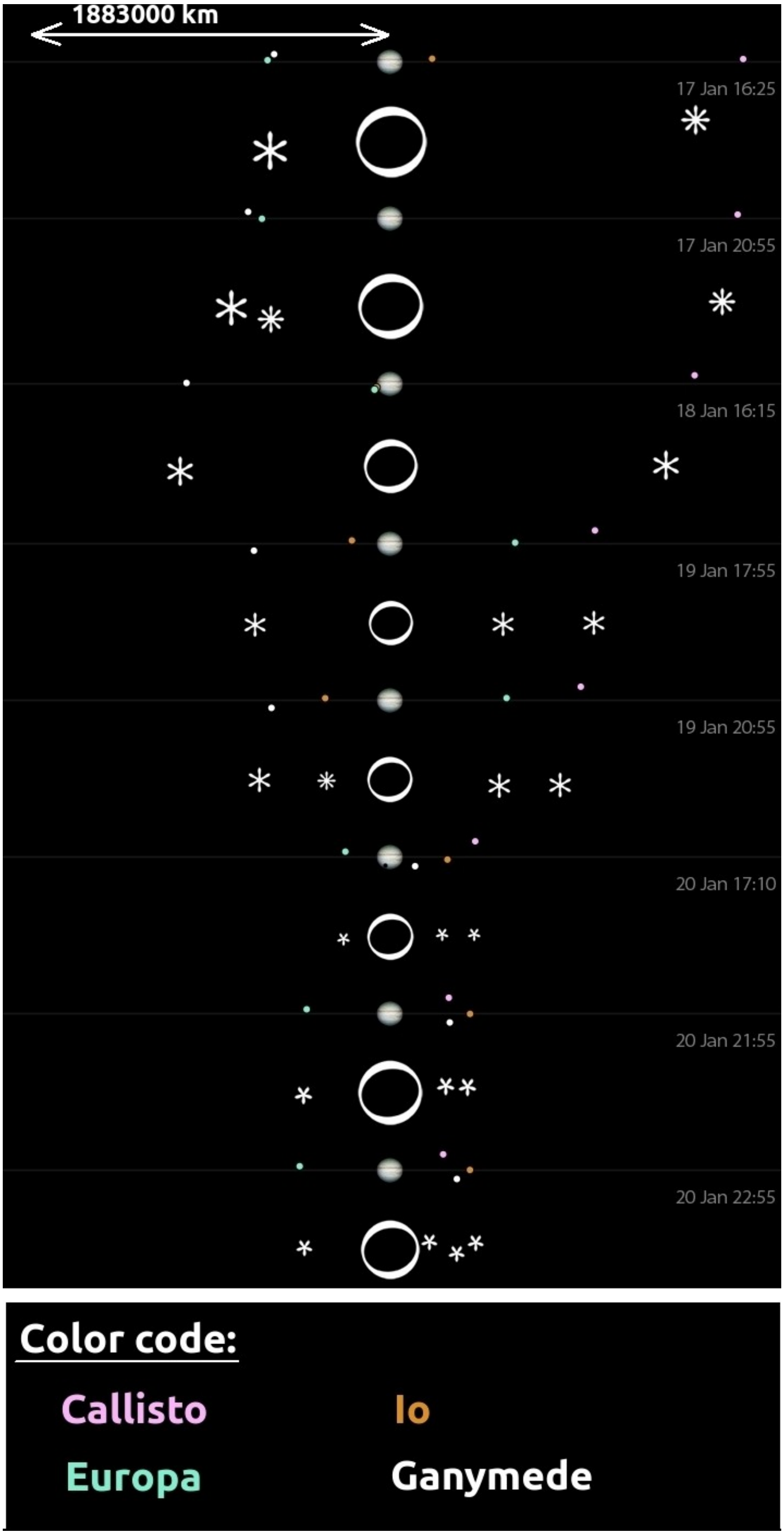
Part 1. Orbital period of one of Jupiter's moons

Over the course of several days, Galileo observed and wrote how Jupiter's moons changed position. Below you can see his observations (asterisks represent the moons), in addition to a simulation of how the moons actually looked, so that you can take measurements:

Next page: A*daptation of Sidereus Nuncius. Galileo Galilei. Images by Ernie Wright*[4]

4 http://www.etwright.org/astro/sidnunj.html


1883000 km
07 Jan 16:40
08 Jan 16:45
10 Jan 16:45
11 Jan 16:45
12 Jan 16:50
12 Jan 18:50
13 Jan 16:50
15 Jan 18:50
15 Jan 22:50
16 Jan 16:50


1883000 km
17 Jan 16:25
17 Jan 20:55
18 Jan 16:15
19 Jan 17:55
19 Jan 20:55
20 Jan 17:10
20 Jan 21:55
20 Jan 22:55
Color code:
Callisto
Io
Europa
Ganymede

Chose **one** moon and calculate its orbital period

Result:
- Write the time in International System Units (seconds) -

Part 2. Satellite velocity

Because the satellites' motion is approximately uniform circular movement (UCM), we can find the angular velocity ω using the following formula:

$$\omega = \frac{2\pi}{T}$$

where *T* is the orbital period. The satellite's velocity will then be

$$v = \omega R$$

where *R* is the orbit's radius. To find out the actual distance to the satellite we have to:

a) Look at the image and find the diameter of the orbit of the satellite that is farthest from Jupiter[5].

b) Look for your moon on the day that it's farthest from Jupiter.

c) Relate the diameter with your orbit's radius using the ruler, and finally find out the orbit's radius with a simple proportionality rule.

5 Aunque podemos leer el dato en la imagen anterior, el *caso 4* (ver más adelante) ofrece las pistas para poder calcular esta distancia.

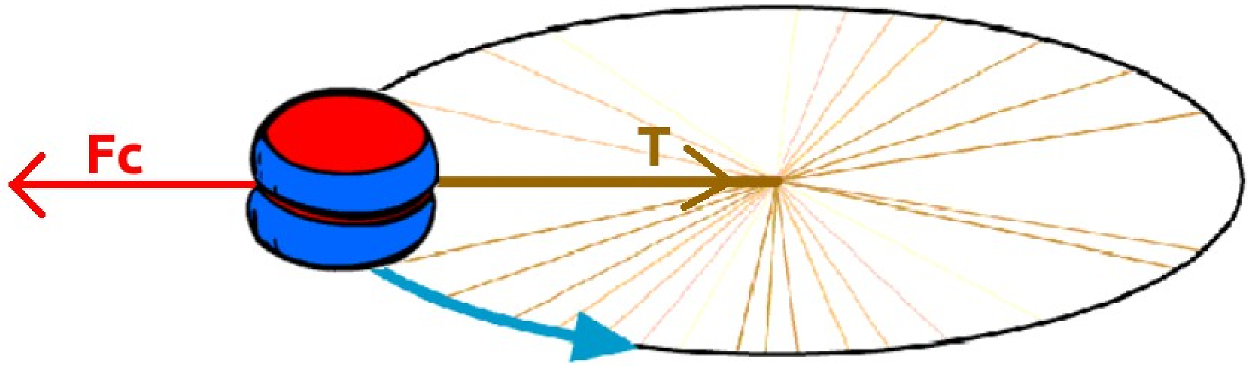
Find out your satellite's velocity.

Result:

- Use International System units -

Part 3. Mass of Jupiter

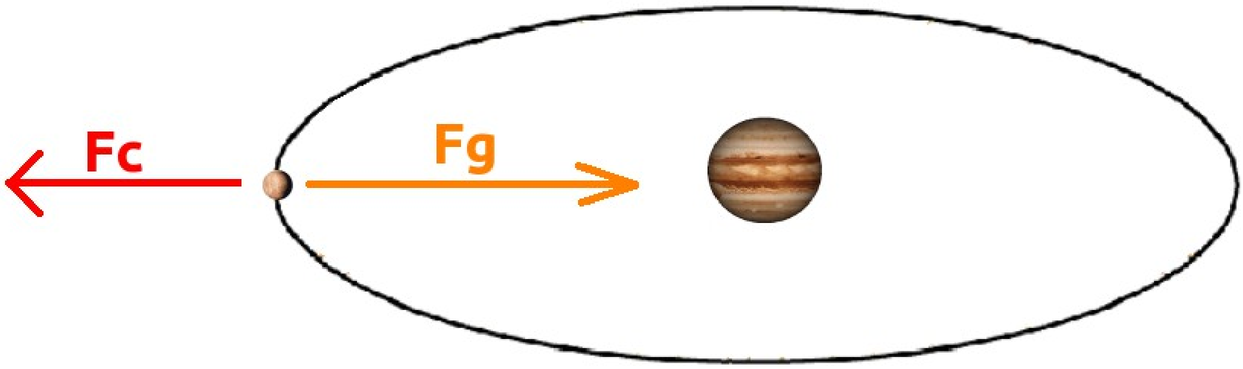
Remember the yo-yo's exercise:



Note that the force or strength experienced by the string is equal to the centrifugal force with which the yo-yo is spinning:

$$T = F_{centrifugal} \quad \rightarrow \quad F_{centrifugal} = \frac{mv^2}{R}$$

Similarly, we can imagine the motion of a moon around its planet:



The gravitational force between a planet and its moon is equivalent to the centrifugal force experienced by the moon!

$$F_{centrifugal} = \frac{M_{Moon} v_{Moon}^2}{R_{Jupiter-Moon}} \quad ; \quad F_{gravitational} = G \frac{M_{Jupiter} M_{Moon}}{R_{Jupiter-Moon}^2} \quad \rightarrow \quad F_{centrifugal} = F_{gravitational}$$

Donde, $G = 6.67384 \times 10^{-11} \frac{\mathrm{N} \cdot \mathrm{m}^2}{\mathrm{kg}^2}$.

If we rearrange the terms correctly, we can find out the mass of Jupiter from the known data.

Find out the mass of Jupiter

Result:

- Use International System Units -

Conclusions and new questions

Why can we find out the mass of Jupiter using any of its moons?

Are the formulas valid for any mass? Justify your answer

Is the attraction we feel towards Earth of the same nature as the attraction that exists between celestial objects?

Further: What is the relationship between gravity and the cause of ocean tides?

Case 2: Practical exercise to do in class

<<***J**upiter* ***Ic**y* **Moons** ***E**xplorer* **(JUICE)** is a special mission proposal by the European Space Agency (ESA) with the aim of developing a space probe to study Jupiter and its moons, especially Ganymede and Europa. >>[6]

Make an educated guess about the orbital radius, velocity and period that JUICE could have in order to orbit around one of these moons. Read up on any sources you wish to get any data you need. You will have to justifiedly decide on the value of one of the three variables (radius, velocity, or period), and calculate the other two.

Further: If you are interested, you can go deeper on your study by using possible elliptical orbits. You cand find more on satellite orbits here: http://www.esa.int/Our_Activities/Space_Science/Types_of_orbit .

6 Source: https://en.wikipedia.org/wiki/Jupiter_Icy_Moon_Explorer

Case 3: Practical kinematics exercise to do in class[7]

Groups of four are recommended.

1. Measure the mass of a small object, like a yo-yo..
2. Tie it to a string of known length, for instance, 0.5 metres.
3. Make it spin horizontally with an easily measurable period, like T = 1 s.

Calculate:

a) Normal acceleration, $\boldsymbol{a_n}$*,*

a) Tension of the string

Calculate and measure:

c) Distance at which the object falls.

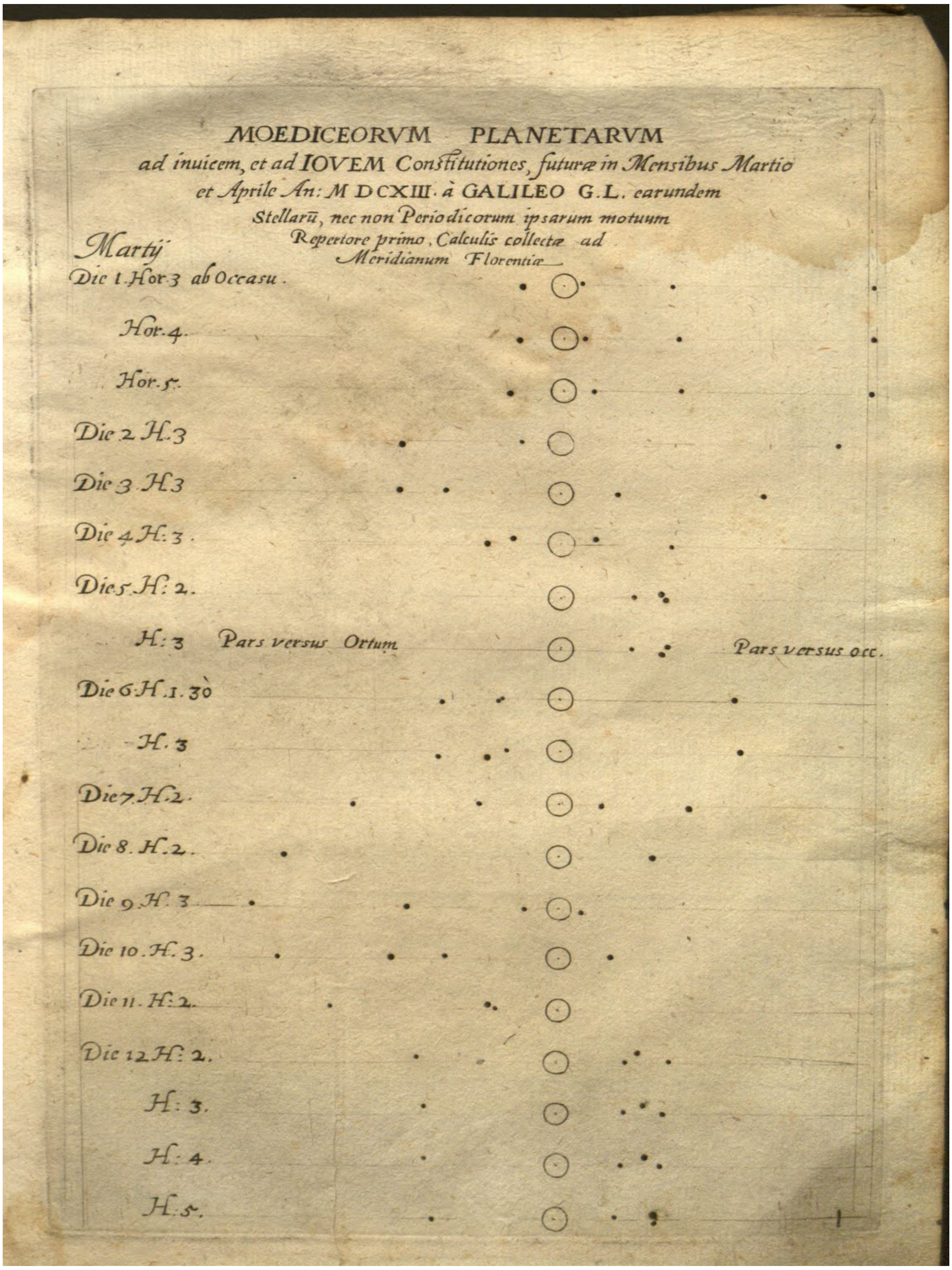
Compare your results with your theoretical data.

Research material:

- Stopwatch.
- Ruler or tape measure.
- Pen and paper.
- Calculator.

7 Exercise about *uniform circular motion* (u.c.m), *projectile motion*, and circular motion dynamics.

First satellite observations by Galileo Galilei.



MOEDICEORVM PLANETARVM

ad inuicem, et ad IOVEM Constitutiones, futuræ in Mensibus Martio et Aprile An: M DCXIII. à GALILEO G.L. earundem Stellarũ, nec non Periodicorum ipsarum motuum Repertore primo, Calculis collectæ ad Meridianum Florentiæ

Martij

Die 1. Hor. 3 ab Occasu.

Hor. 4.

Hor. 5.

Die 2 H. 3

Die 3 H 3

Die 4 H: 3.

Die 5 H: 2.

H: 3 Pars versus Ortum — Pars versus occ.

Die 6 H. 1. 30

H. 3

Die 7 H. 2.

Die 8. H. 2.

Die 9 H. 3

Die 10. H. 3.

Die 11. H. 2.

Die 12 H: 2.

H: 3.

H: 4.

H: 5.

Extract from Sidereus Nuncius, written by Galileo Galilei in 1610.