CESAR Science Case

The mass of Jupiter

Calculating the mass of a planet from the motion of its moons

Teacher Guide

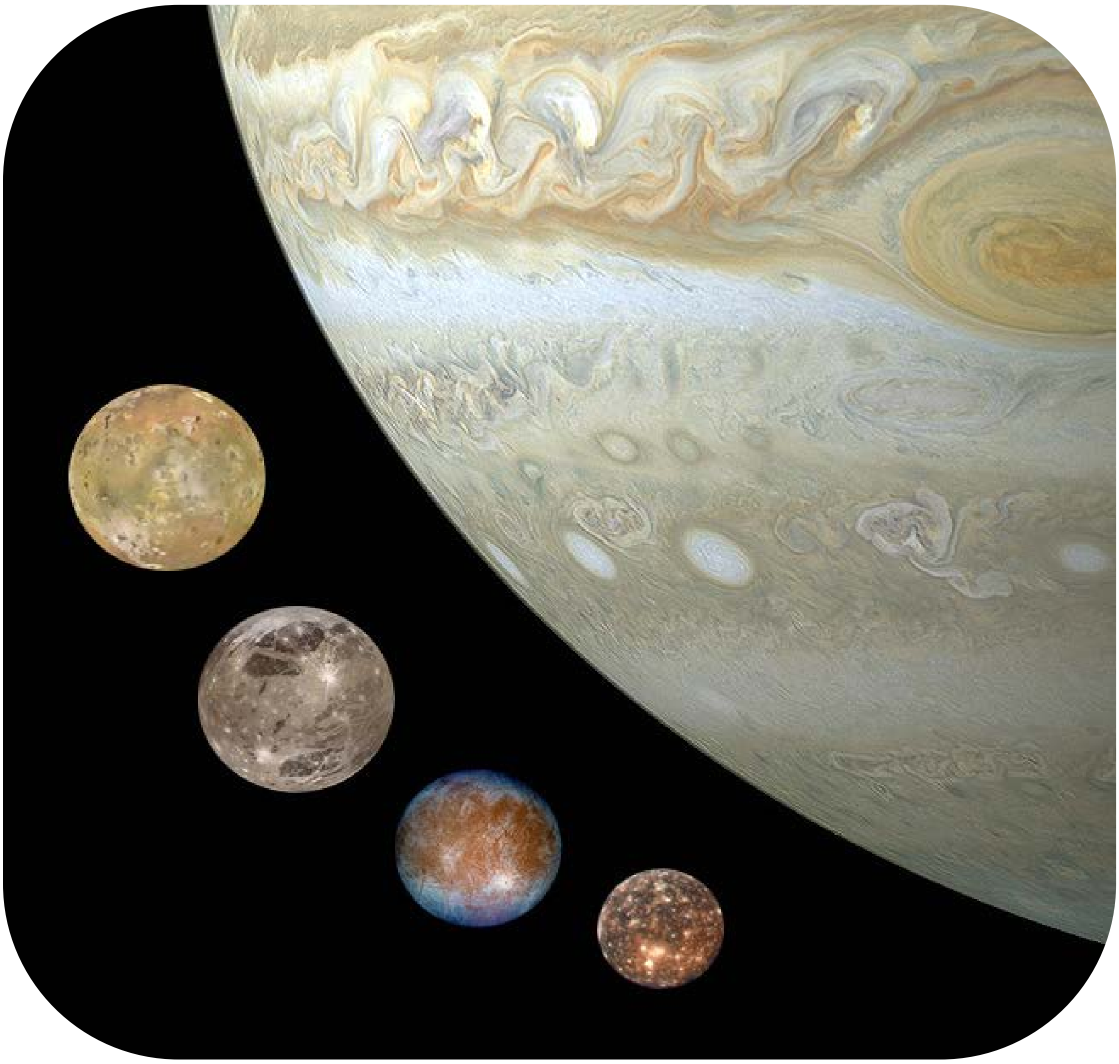

Table of Contents

Fast Facts

FAST FACTS

Age range: 16 - 18 years old

Type: Student activity

Complexity: Medium

Teacher preparation time: 30 - 45 minutes

Lesson time required: 1 hour 45 minutes

Location: Indoors

Includes use of: Computers, internet

Curriculum relevance

General

- Working scientifically.
- Use of ICT.

Physics

- Kepler's Laws
- Circular motion
- Eclipses

Space/Astronomy

- Research and exploration of the Universe.
- The Solar System
- Orbits

You will also need…

Computer with required software installed:

http://stellarium.org

www.cosmos.esa.int/web/spice/cosmographia

To know more…

CESAR Booklets:

- Planets
- Stellarium
- Cosmographia

Outline

In these activities students find out about the moons of Jupiter and measure their main orbital parameters. Students will then use this information and apply their knowledge about the orbits of celestial bodies to calculate the mass of the planet Jupiter.

Students should already know…

- Orbital Mechanics (velocity, distance…)
- Kepler's Laws
- Trigonometry
- Units conversion

Students will learn…

- How to apply theoretical knowledge to astronomical situations.
- The basics of astronomy software.
- How to make valid and scientific measurements.
- How to predict astronomical events.

Students will improve…

- Their understanding of scientific thinking.
- Their strategies of working scientifically.
- Their teamwork and communication skills.
- Their evaluation skills.
- Their ability to apply theoretical knowledge to real-life situations.
- Their skills in the use of ICT.

Summary of activities

Title	Activity	Outcomes	Requirements	Time
1. Choose your moon	Students choose their favourite moon of Jupiter using *Cosmographia.*	Students improve: • Thinking and working scientifically. • ICT skills.	• *Cosmographia* installed. Step by step Installation guide can be found in: Cosmographia Booklet.	20 min
2. Calculate the mass of Jupiter	Students use the *Stellarium* software to calculate the orbital period and radius of a Jupiter moon. They then use these parameters to calculate the mass of Jupiter.	Students improve: • The first steps in the scientific method. • Working scientifically • ICT skills. • Applying theoretical knowledge. Students learn: • How astronomers apply calculus.	• Completion of Activity 1. • *Stellarium* installed. Step by step Installation guide can be found in: *Stellarium* Booklet.	1 hour
Extension: activity: Predict a Transit	Students analyse the motion by another method, using uniformly accelerated motion equations.	Students learn: • Application of calculus using real data. • Basic properties of a star. • What information can be seen and extracted from an astronomical image. Students improve: • Thinking and working scientifically. • Teamwork and communication skills. • Application of theoretical knowledge to real-life situations. • ICT skills.	• Completion of Activities 1 and 2.	25 min

Introduction

The gas giant Jupiter is the largest planet in our Solar System. It doesn't have a proper surface and is made up of swirling clouds of gas and liquids that are mostly hydrogen and helium. Jupiter is so large that about 11 Earth's could fit across it. It is around 320 times heavier than the Earth and its mass is more than twice the mass of the all the other planets in the Solar System combined.

Jupiter has 79 moons (as of 2018) – the highest number of moons in the Solar System. This number includes the Galilean moons: Io, Europa, Ganymede, and Callisto. These are Jupiter's largest moons and were the first four to be discovered beyond Earth by astronomer Galileo Galilei in 1610.

By measuring the period and the radius of a moon's orbit it is possible to calculate the mass of a planet using Kepler's third law and Newton's law of universal gravitation. In these activities students will make use of these laws to calculate the mass of Jupiter with the aid of the *Stellarium* (stellarium.org) astronomical software. Prior to this they explore the Galilean moons using a 3D visualisation tool, *Cosmographia, (www.cosmos.esa.int/web/spice/cosmographia)*.

The Galilean moons (Io, Europa, Ganymede and Callisto) are distinctive worlds of their own and of high scientific interest.

- **Io**: The most volcanically active object in all the Solar System due to the inward gravitational pull from Jupiter and the outward pull from other Galilean moons.
- **Europa**: A cold world that might have a liquid water ocean beneath a thick layer of surface ice. Of Jupiter's moons, Europa is the one scientists believe is more likely to be habitable.
- **Ganymede**: The largest known moon. There is evidence that it conceals a liquid water ocean under its icy shell; potentially an environment suitable for life.
- **Callisto**: Has an old and heavily cratered surface, therefore providing a window to explore the early formation of the moons. Also, thought to have an ocean beneath the surface.

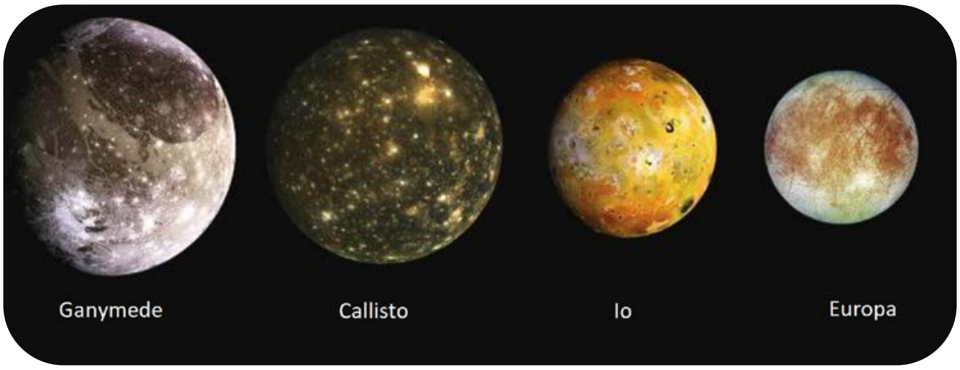

Figure 1: The Galilean moons (Credit: NASA)

The JUICE - JUpiter ICy moons Explorer – mission is planned for launch in 2022 and arrival at Jupiter in 2029, it will spend at least three years making detailed observations of Jupiter and

Ganymede, Callisto and Europa. The focus of JUICE is to characterise the conditions that may have led to the emergence of habitable environments among the Jovian icy satellites.

Background

Kepler's Laws, published between 1609 and 1619, led to a huge revolution in the 17th century. With them scientists were able to make very accurate predictions of the motion of the planets, changing drastically the **geocentric model of Ptolomeo** (who claimed that the Earth was the centre of the Universe) and the **heliocentric model of Copernicus** (where the Sun was the centre but the orbits of the planets were perfectly circular). These laws can also explain the movement of other Solar System bodies, such as comets and asteroids.

Kepler's laws can be summarised as follows:

1. *First Law:* The orbit of every planet is an ellipse, with the Sun at one of the two foci.
2. *Second Law:* A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.

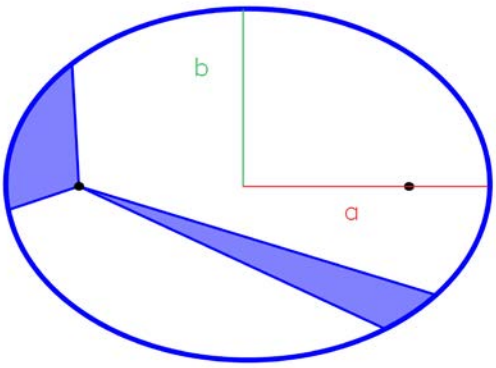

Figure 2: Second Law of Kepler (Credit: Wikipedia)

3. *Third Law:* The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.

Assuming that a planet moves in a circular orbit with no friction, the gravitational force, F_G, equals the centrifugal force, F_C, Kepler's third law can therefore be expressed as:

$$F_G = F_C \quad \rightarrow \quad \frac{GMm}{R^2} = m\, a_c$$

$$as \quad a_c = \frac{v^2}{R} \quad \rightarrow \quad \frac{GMm}{R^2} = m\frac{v^2}{R}$$

$$and\ as \quad v = \omega \cdot R = \frac{2\pi}{T} R \quad \rightarrow \quad \frac{GM}{4\pi^2} = \frac{R^3}{T^2}$$

Where, M, is the mass of a planet and, m, is the mass of an orbiting moon. For the moon, v, is the linear velocity (in metres per second), R, is the radius of its orbit (in metres), ω, is the angular velocity of the moon (expressed in radians per second), T, is the orbital period (in seconds) and, G, is the universal gravitational constant, with a value of $G = 6.674 \cdot 10^{-11}\, m^3\, kg^{-1}\, s^{-2}$

Therefore, $T^2 \propto R^3$ is achieved as follows:

$$\frac{GM}{4\pi^2} = \frac{R^3}{T^2} \quad \rightarrow \quad \boxed{M_J = \frac{4\pi^2}{G}\frac{R^3}{T^2}}$$

Activity 1: Choose your moon

In this activity students use the *Cosmographia* software to find out more about Jupiter's largest moons and choose which moon they would like to use to calculate the mass of Jupiter.

Full instructions are provided in the Student Guide.

The students are asked to complete a table with physical information about the different moons, a completed version can be found in Table 1.

Object	Mass (kg)	Radius (km)	Density (g/cm^3)
Jupiter	$1.8982 \cdot 10^{27}$	69 911	1.326
Io	$8.9319 \cdot 10^{22}$	1 824	3.53
Europa	$4.8000 \cdot 10^{22}$	1 563	3.01
Ganymede	$1.4819 \cdot 10^{23}$	2 632	1.94
Callisto	$1.07594 \cdot 10^{23}$	2 409	1.84

Table 1: Table of physical properties of Jupiter and the Galilean moons with key

Activity 2: Calculate the mass of Jupiter

In this activity students will use the *Stellarium* software to calculate the orbital period and radius of their favourite Galilean moon, as chosen in Activity 1. They will then use these parameters to calculate the mass of Jupiter.

2.1 Calculate the orbital period of your moon

To begin the students use *Stellarium* to calculate the orbital period of their moon.

Full instructions are provided in the Student Guide.

The periods of all the Galilean moons can be found in Table 2.

Moon	Orbital period
Io	1 day 18.45 hours
Europa	3 days 12.26 hours
Ganymede	7 days 3.71 hours
Callisto	16 days 16.53 hours

Table 2: Period of the Galilean moons

An example of the calculation is as follows:

Your moon	**Europa**

Initial date (YYYY-MM-DD hh:mm:ss)	Final date (YYYY-MM-DD hh:mm:ss)
2018-09-01 03:05:00	**2018-09-04 15:25:00**

Calculate the time difference here

Same year and same month

4th – 1st = 3 days

15h – 3h = 12 h

25 min – 05 min = 20 min

And as 1h = 60 min → 20 min = 0.3 h

Period	**3**	days	**12.3**	hours

The students can also play with the time rate in *Cosmographia* and check their result for the period of their moon by visualising the motion in 3D.

2.2 Calculate the orbital radius of your favourite moon

Next, the students need to calculate the radius of the orbit of their moon, as Kepler's third law involves this term. And, as explained in the Stellarium booklet, the "Angle Measure" plugin needs to be enabled.

The students will use trigonometry to calculate the relationship between angular distance, θ, and the orbital distance of every moon, R, The distance from Jupiter to Earth, d_{JE}, can be obtained using *Stellarium*.

Again, as an example, using the previous results:

Maximum distance of your moon to Jupiter	**0° 2' 40.31''**	**0.0445°**

$d_{JE} =$	**5.72** au	**8.55 · 10⁸** km

$R = d_{JE} \sin\theta$

R = 8.55 · 10⁸ sin (0.0445 º) = 664 761 km

For metres, multiply by 10^3

$R =$	**664 761** km	**6.64 · 10^8** m

$$v = \omega \cdot R = \frac{2\pi}{T} R$$

$$T = 3\text{ d } 12.3\text{h} = 3\cdot 24 + 12.3\text{ h} = 84.3\text{ h} = 84.3\text{ h}\cdot\frac{3600\text{ s}}{1\text{ h}} = 303\,480\text{ s}$$

$$v = \frac{2\pi}{303480}\, 6.64\cdot 10^8 = 13.76\cdot 10^3\text{ m/s} = 13\,760\text{ m/s}$$

$v =$	**13 763** m/s

With this information, both the orbital radius and velocity can be calculated.

Moon	Orbital radius (km) (Semi-major Axis)	Orbital velocity (m/s)
Io	421 700	17 334
Europa	670 900	13 740
Ganymede	1 070 400	10 880
Callisto	1 882 700	8 204

Table 3: Chart with orbital radius and velocity for each Galilean moon

No solution is provided for the angular distance, θ, since it will depend on the distance from Earth to Jupiter, which is not always the same.

To check if the measurement has been made correctly the students must calculate, R, (the distance from Jupiter to the moon) and then this result must be compared to the real values given in Table 3. The students value may differ slightly due to **errors** in the measurements. An error of less than or equal to 5% is be acceptable. The same goes for the value of the velocity.

To calculate the relative error for any measurement:

$$E_R = \frac{|\,Measured\ Value - Real\ Value\,|}{Real\ Value}\ .100\ (2)$$

$$\rightarrow\ E_R = \frac{|\ 664\ 761 - 670\ 900\ |}{670\ 900} \cdot 100 = \frac{6139}{670\ 900} \cdot 100 = 0.92\%$$

Note: A negative value for the relative error will probably mean that the absolute value of equation (2) has not been applied.

2.3 Calculate the Mass of Jupiter

The most accurate value for the mass of Jupiter is

$M_J =$	$1.8982 \cdot 10^{27}\quad kg$

Applying Kepler´s third Law:

$$\frac{GM_J}{4\pi^2} = \frac{R^3}{T^2} \qquad \rightarrow \qquad M_J = \frac{4\pi^2}{G}\frac{R^3}{T^2}$$

Using the values for the orbital radius and orbital period obtained in the previous examples (sections 2.1 and 2.2) the mass of Jupiter can be calculated as follows:

$$M_J = \frac{4\pi^2}{G}\frac{R^3}{T^2} = \frac{4\pi^2}{6.674\cdot10^{-11}\ m^3\ kg^{-1}\ s^{-2}} \cdot \frac{\left(6.64\cdot10^{8}\ m\right)^3}{(303480\ s)^2} = 1.8867\cdot10^{27}\ kg$$

Additional Activity: Predict a Transit

In this activity students predict a transit of Jupiter by one of its largest moons using *Stellarium*.

For predicting a future transit the students must first find a previous one. The *Stellarium* software is recommended for this purpose. Adding the following code to the script run in Activity 2:

```
StelMovementMgr.zoomTo(0.0167, 5);
core.setDate("2018:08:17T00:20:50","utc");
core.setTimeRate(300);
```

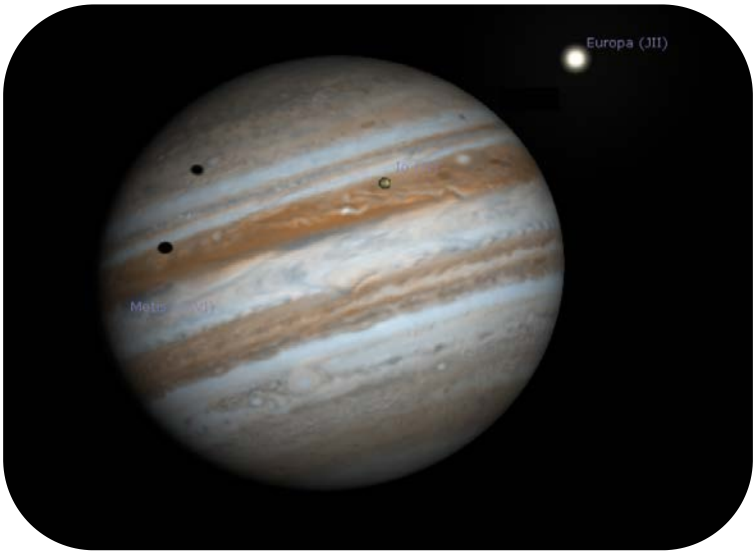

Figure 3: Io and Europa transit, using Stellarium

Jupiter will fill the screen (Figure 3), and the script is already programmed for visualising the transit of Europa and Io. In order to visualise new transits, the students must press on , or number 8 on their keyboard, to adjust the date of *Stellarium* to the current time and date.

Later, the time rate can be changed using the . Each time they press the time rate the speed is multiplied by 10, therefore just touching this button two or three times will be adequate for this activity.

Press to stop the motion. Figure 4 shows the menu for changing the time rate, which is in the lower left part of the screen.

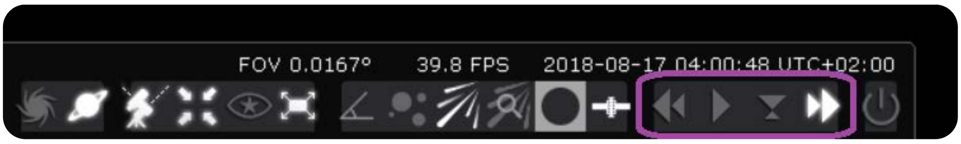

Figure 4: Time rate menu

To predict transits the students should have to hand the orbital period they calculated for their favourite moon. By adding the period to the initial time/end time they will be able to predict when the next transit will start/end.

A transit can be seen as a shadow of the satellite cast on the disk of the planet. For this activity it is not recommended to choose Callisto. This is because it is the furthest Galilean moon and has an inclined orbit. Therefore, often its shadow misses the planet. Figure 5 shows a sketch for orbit inclination.

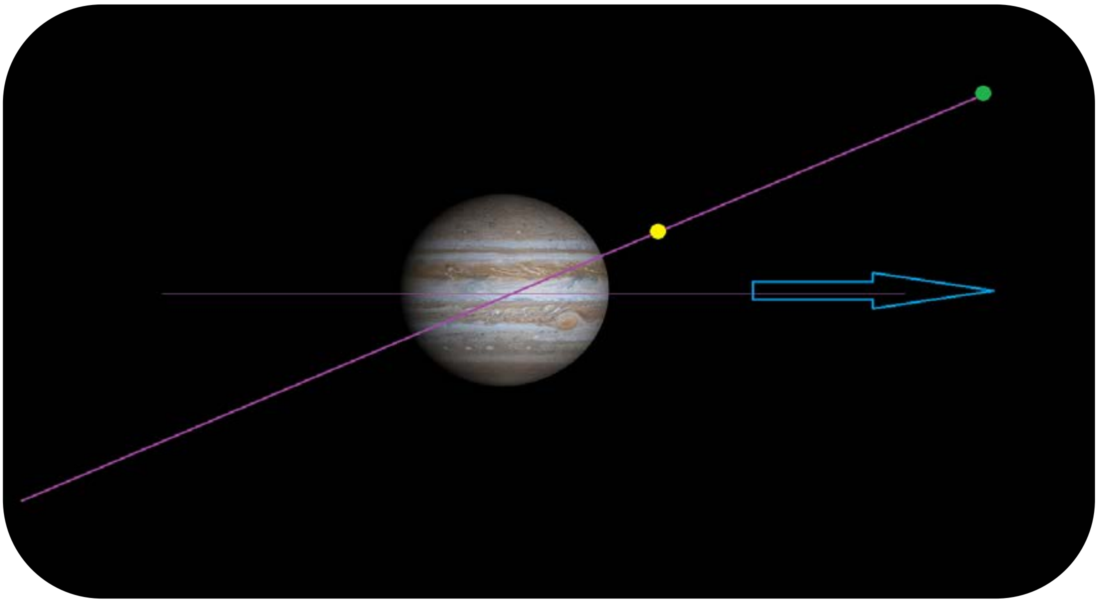

Figure 5: Inclination sketch of an orbit (not to scale). The blue line represents Earth's direction
The yellow moon is close to Jupiter, so the transit could be seen.
The green moon has the same inclination, but as it is further away the transit could not be seen.

Answer to question in the Student Guide

Do you think the transit could be seen with telescopes on Earth? And with space telescopes? Why?

The transits of Jupiter´s Galilean moons can always be seen with space telescopes. But there are two main reasons why some transits cannot be seen from Earth:

- Optical telescopes on Earth depend on light conditions and therefore only operate at night. So only the transits that can be seen are those at night.

- It also depends on the position of the Earth. The constellations that can be seen in summer are not the same constellations visible at winter. That is because the Earth is orbiting the Sun and the axis of rotation is tilted by 23.4º, so the day and night skies change over the course of the year. The stars and constellations that can be seen during the whole year are called circumpolar.

In conclusion, the orbit of the Earth and the orbit of Jupiter are also factors to take into account.

The students can check if their prediction is correct by entering that date and time into *Stellarium* software and checking if the shadow of the moon appears on Jupiter. This can be achieved by two different ways:

- By console: Open the console by pressing F12 and add the following lines to the code. Change the second line by entering the predicted date and time. Run the script.

```
StelMovementMgr.zoomTo(0.0167, 5);
core.setDate("2018:08:17T00:20:50","utc");
core.setTimeRate(0);
```

- By user interface: Use the buttons shown in Figure 4 to move to the predicted time and date.

Alternatively, a chart for future transits can be found here:
https://www.skyandtelescope.com/wp-content/observing-tools/jupiter_moons/jupiter.html#

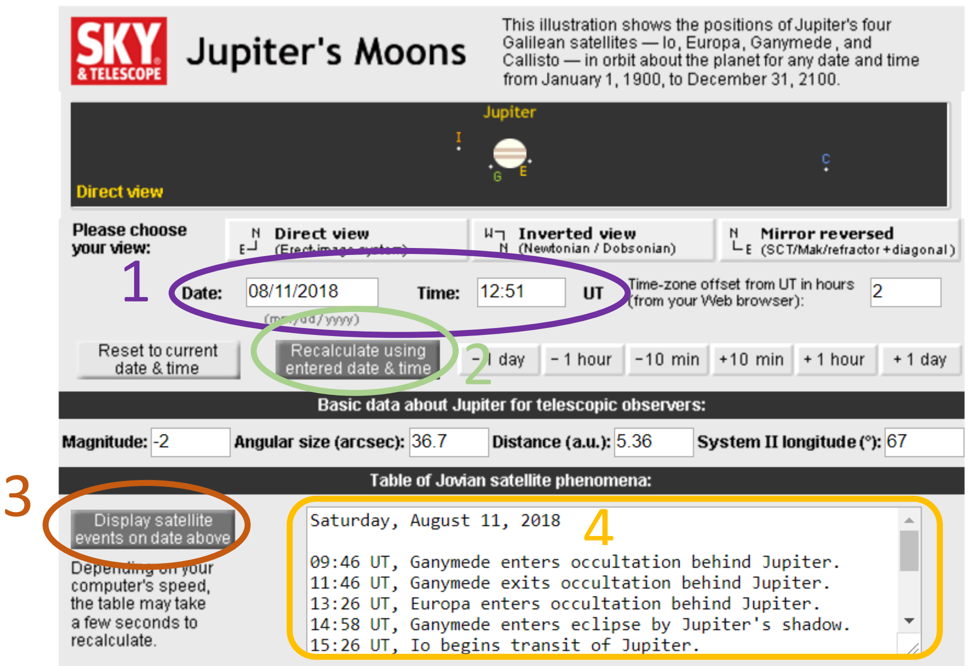

Figure 6: Sky&Telescope Jupiter's transits predictor

You can check if the students have predicted the transit correctly by using the website shown in Figure 6. In order to do that:

- Enter the predicted date and time of the transit in the text boxes labelled number 1 in Figure 6.
- Click on “Recalculate using entered date and time” labelled 2, to have a representation of the moons position at that time.
- Click on “Display satellite events on date above” labelled 3, and all the information will be displayed in the textbox labelled 4.

Links

Software

- Cosmographia download: www.cosmos.esa.int/web/spice/cosmographia
- CESAR Booklet: *Cosmographia*

- Cosmographia Official Users guide
  https://cosmoguide.org/

- CESAR Booklet: *Stellarium*

- Stellarium Official Users Guide
  https://github.com/Stellarium/stellarium/releases/download/v0.18.1/stellarium_user_guide-0.18.1-2.pdf

Planets

- CESAR Booklet: *Planets*
- JUICE mission: http://sci.esa.int/juice/

Kepler's Laws

- CESAR Science Case: *Orbits (Spanish only)*

- Kepler's Laws Animation
  http://astro.unl.edu/classaction/animations/renaissance/kepler.html