CESAR Science Case

The mass of Jupiter

Calculating the mass of a planet from the motion of its moons

Student Guide

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Introduction

The gas giant Jupiter is the largest planet in our Solar System. It doesn't have a proper surface and is made up of swirling clouds of gas and liquids that are mostly hydrogen and helium. Jupiter is so large that about 11 Earth's could fit across it. It is around 320 times heavier than the Earth and its mass is more than twice the mass of the all the other planets in the Solar System combined.

Jupiter has 79 moons (as of 2018) – the highest number of moons in the Solar System. This number includes the Galilean moons: Io, Europa, Ganymede, and Callisto. These are Jupiter's largest moons and were the first four to be discovered beyond Earth by astronomer Galileo Galilei in 1610.

Figure 1: Galilean moons

By studying the motion of one of these moons it is possible to calculate the mass of Jupiter, just by applying Kepler's third law and Newton's law of universal gravitation. Kepler's Laws were published between 1609 and 1619, and led to a huge revolution in the 17th century. With them scientists were able to make very accurate predictions of the motion of the planets, changing drastically the **geocentric model of Ptolomeo** (who claimed that the Earth was the centre of the Universe) and the **heliocentric model of Copernicus** (where the Sun was the centre but the orbits of the planets were perfectly circular). These laws can also explain the movement of other Solar System bodies, such as comets and asteroids.

Activity 1: Choose your moon

In this activity you will use the astronomical software *Cosmographia* to find out more about Jupiter's largest moons. You will then choose which moon you would like to use to calculate the mass of Jupiter.

1. Double click on "Cosmographia" and the application will open as shown in Figure 2.

Figure 2: Cosmographia starting view

2. Select the top icon in the menu on the left-hand side of the screen (the white circles). Images of various Solar System bodies will be displayed. See Figure 3.

Figure 3: Solar System bodies as seen in Cosmographia

3. Find and click on the image of Jupiter. The application will then take you to Jupiter.

4. Adjust the field of view (zoom out or in) until you can see all four Galilean moons (Io, Europa, Ganymede and Callisto), as shown in Figure 4. This can be done either by holding down shift and dragging the mouse to the right, or using a pinch gesture on a track pad.

Note: The default settings of Cosmographia show the positions of Solar System objects in real-time therefore the moons will most likely be in different positions to Figure 4.

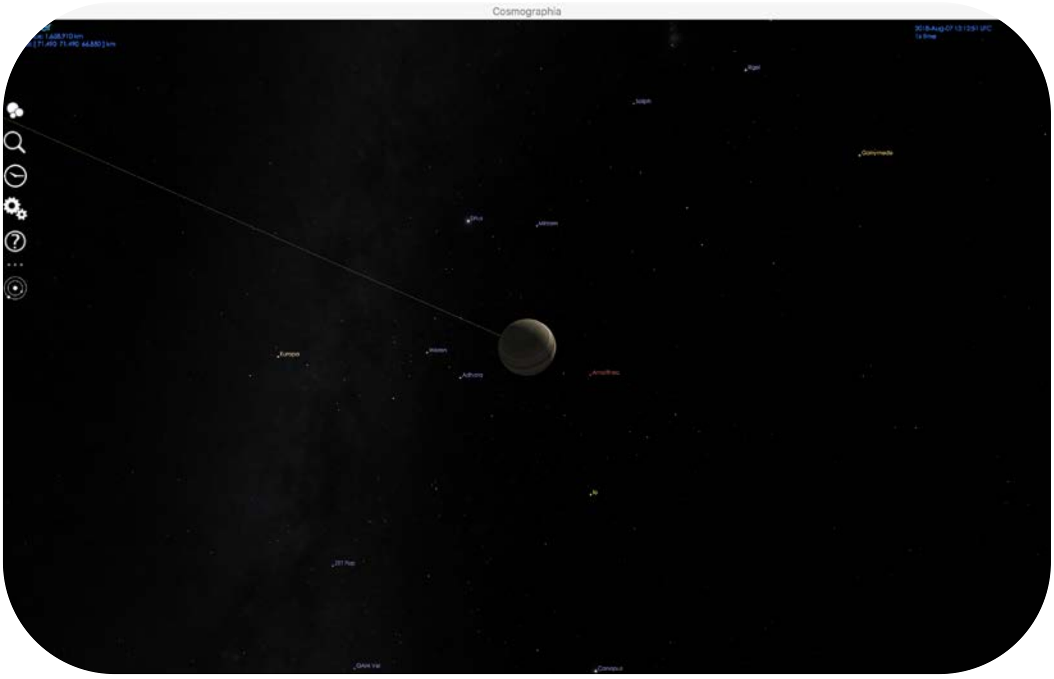

Figure 4: Solar System bodies as seen in Cosmographia

5. To find out more information about each moon, right click on each one in turn to display a menu of options, as shown in Figure 5. ‘Show properties’ at the bottom of this menu provides physical information about the object (Figure 6).

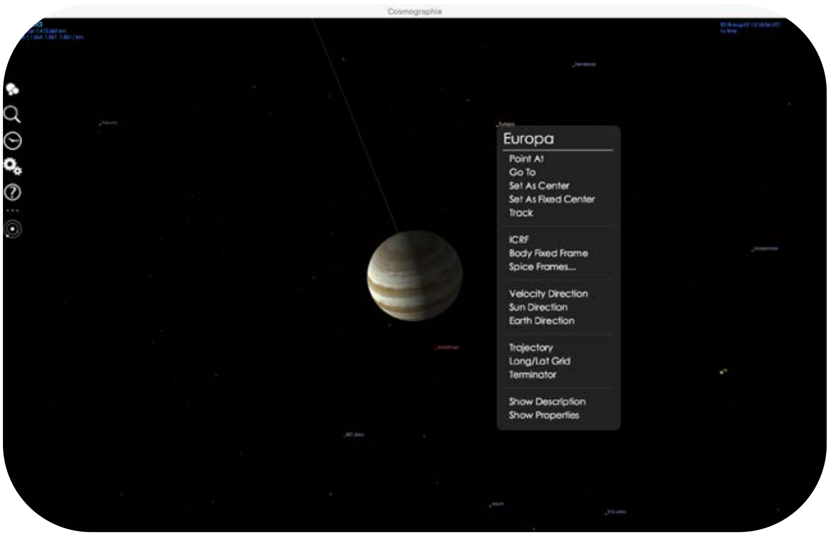

Figure 5: Trajectory toggle menu as seen in Cosmographia

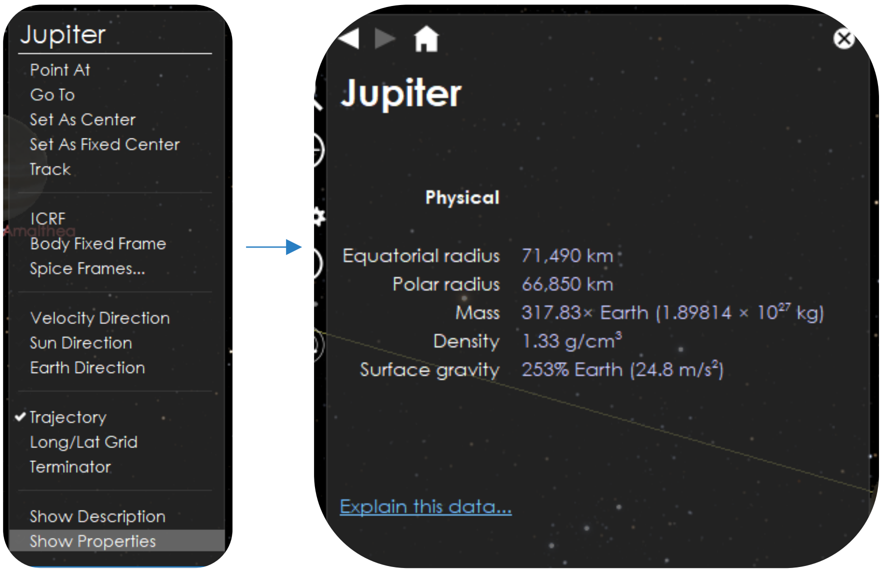

Figure 6: Right click on Jupiter (left image); Jupiter properties (right image) as seen in Cosmographia

6. Use the information about the physical properties of each moon and of Jupiter to complete Table 1.

Object	Mass (kg)	Radius (km)	Density (g/cm^3)
Jupiter			
Io			
Europa			
Ganymede			
Callisto			

Table 1: Table of physical properties of Jupiter and the Galilean moons

7. Now that you are familiar with the Galilean moons, choose your favourite to use in the Activity 2 to calculate the mass of Jupiter.

Your moon	

Activity 2: Calculate the mass of Jupiter

In this activity you will use Kepler's third law to calculate the mass of Jupiter. You will find and work out the different variables needed for your calculations using a piece of astronomical software called *Stellarium*.

2.1 Calculate the orbital period of your moon

Next you will use *Stellarium* to calculate the period of your chosen moon.

8. Open *Stellarium*, and then open the console by pressing F12. Copy and paste the script given in Figure 7 This script tells *Stellarium* to travel to Jupiter, and configures the program as needed for the activity. You can toggle to and from Full view using F11.

```
core.setObserverLocation("Madrid, Spain");

LandscapeMgr.setFlagLandscape(false);
LandscapeMgr.setFlagAtmosphere(false);
LandscapeMgr.setFlagFog(false);

core.selectObjectByName("Jupiter", true);
core.setMountMode("equatorial");
core.setTimeRate(3000);
StelMovementMgr.setFlagTracking(true);
StelMovementMgr.zoomTo(0.167, 5);
```

Figure 7: Script to copy into Stellarium console to view Jupiter close up

9. Once you have done this click on the play button ( ). The view on your screen will look similar to Figure 8.

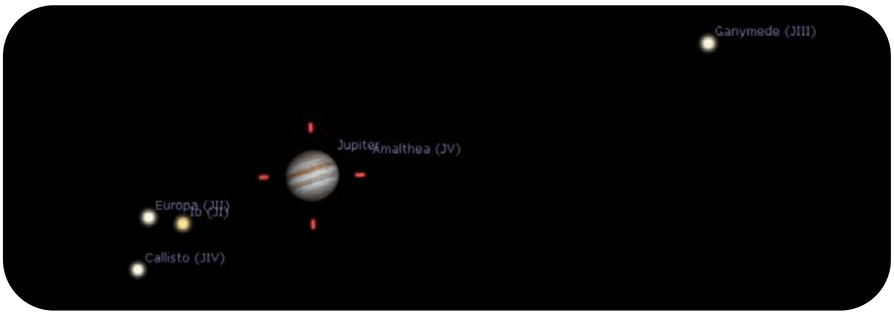

Figure 8: Stellarium view, after running the script

10. In this view the Galilean moons should be labelled. Find your chosen moon and watch how it orbits Jupiter. It is following a periodic motion. However, note that its real motion is about 3000 times slower.

You next need to calculate how long your moon takes to make a complete orbit.

11. Slow down the motion of the moons and pay attention to the “date” and “time” parameters in the lower part of the display. Select a starting point and write down the date and time in Table 2. Watch the moon make one complete orbit and note the date and time that the moon returns to the same position. The difference between these two dates and times is the period.

Initial date (YYYY-MM-DD hh:mm:ss)	**Final date (YYYY-MM-DD hh:mm:ss)**
Calculate the time difference here	

Table 2: Moon period calculation

12. Write down the period of your moon here:

Period	days	hours

Note: In *Stellarium* we see Jupiter as it is seen from Earth. However, the orbital motion of the moons is actually circular, we are simply viewing 2 dimensional projection.

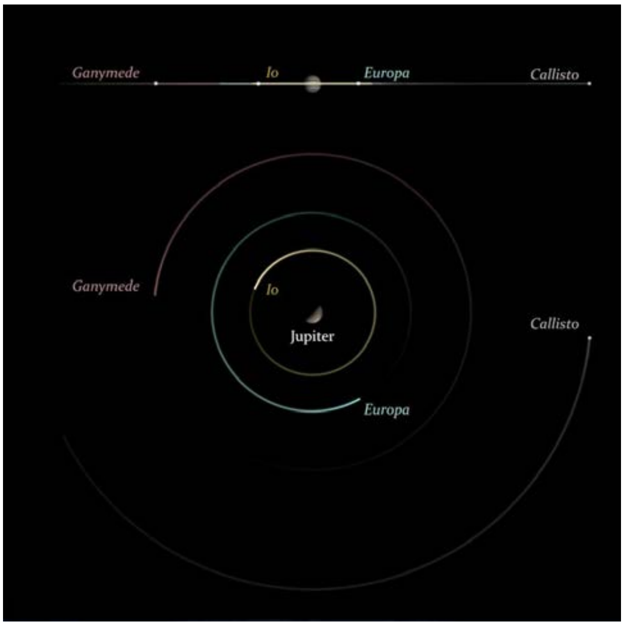

Figure 9: Visualisation of the Jupiter moons (Credit: CESAR)

Did you know?

Jupiter has always been a very interesting astronomical object to study. Since the first observations were made by Galileo using an early version of the telescope huge advancements in technology have been achieved.

Several space missions have flown by close to Jupiter, including **Pioneer 10**, **Pioneer 11**, **Voyager 1, Voyager 2** and **Cassini-Huygens**. In addition, other missions have been developed to specifically study Jupiter and its moons. In 1995 NASA's Galileo spacecraft became the first to orbit Jupiter. NASA's **JU**NO spacecraft arrived at the planet in 2016 for a 5 year mission. ESA is currently working on the JUpiter Icy moon Explorer (**JUICE)**, which is scheduled for launched in 2022.

Artist's impression of the JUICE mission exploring the Jupiter system. (Credit: Spacecraft: ESA/ATG medialab; Jupiter: NASA/ESA/J. Nichols (University of Leicester); Ganymede: NASA/JPL; Io: NASA/JPL/University of Arizona; Callisto and Europa: NASA/JPL/DLR)

2.2 Calculate the orbital radius of your moon

The orbital radius of your moon can be defined as the maximum distance between Jupiter and the moon, assuming that the orbit of your moon around Jupiter is circular. As you can see in Figure 11, the orbital radius can be obtained using trigonometry.

To calculate the orbital radius you will use *Stellarium* with the Angle Measure plug-in.

Note: Make sure that the "*Angle Measure*" plug-in is active in your configuration of *Stellarium*. To activate this plug-in follow the steps below:

- Move your mouse to the left part of the screen to view the side menu bar.
- Open the configuration menu (or F2 in your keyboard).
- Go to Plug-ins, select Angle Measure and check the box next to 'Load at startup' at the bottom of the window.
- Restart *Stellarium*.

13. Stop the motion of the moons around Jupiter by clicking on in the menu at the bottom of the screen (or K on your keyboard).

14. In the same menu click on (or press Ctrl + A) to enable the Angle Measuring plugin.

15. Click on the centre of Jupiter and then without releasing drag the curser to the centre of your chosen moon.

Note: Kepler's Laws are only valid if the measurement is made from the centre of both objects, so make sure that you click on the centre of the moon and on the centre Jupiter.

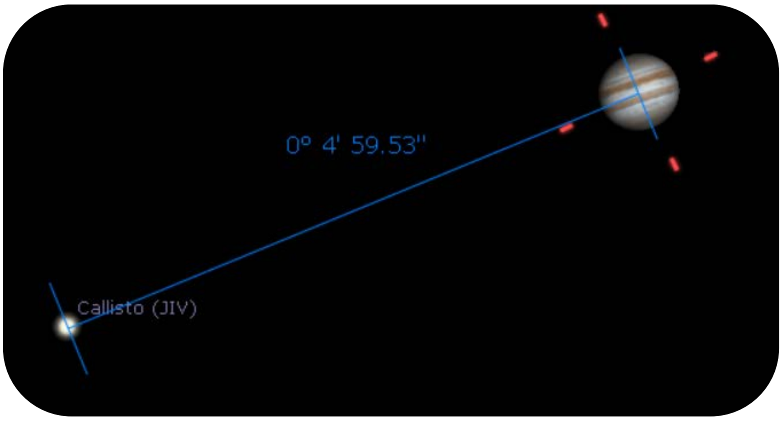

Figure 10: Using Angle Measure plugin

16. Record your measurement in column 2 of Table 3 using the units, degrees, arc minutes, arc seconds, as given in *Stellarium*.

17. As we are considering angular distance this distance needs to be in degrees. Convert these units to degrees (including decimals places) and write down the distance (in degrees).

Maximum Distance of your Moon to Jupiter	° ' ''	°

Table 3: Calculate the distance between Jupiter and your moon

Write your calculations here *Remember: 1° = 60' and 1'= 60'' → 1°= 3600 ''*

18. Record the distance from Earth to Jupiter, d_{JE}, (shown in the information on the left-hand side of the screen when Jupiter is selected). This value is given in au, astronomical units, therefore you will need to convert it to kilometres.

Note: 1 au is the mean distance from the Earth to the Sun, $1\ au = 149\ 584\ 372\ \ km$.

Convert the distance between Earth and Jupiter from astronomical units to kilometres. Write your calculations here.

$d_{JE} =$	au	km

The orbital radius of your moon can now be calculated using trigonometry.

19. Use the diagram shown in Figure 11 to help with your calculation. In Figure 11, the angular distance between Jupiter and the moon is, θ, the distance between Jupiter and Earth is, d_{JE}, and the orbital radius of the moon around Jupiter is, R.

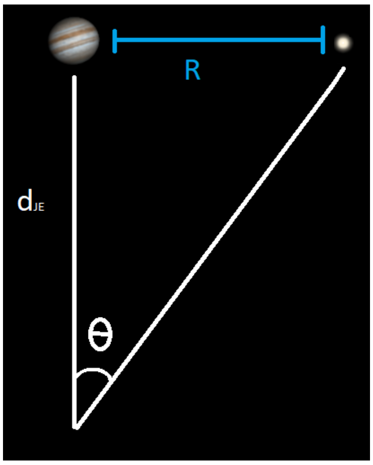

Figure 11: Sketch for calculating moon-Jupiter distance.

Using the trigonometric definition of the tangent of an angle:

$$R = d_{JE} \cdot \tan\theta$$

As the angle is too small, the tangent of the angle can be approximated to its sine, therefore:

$$R = d_{JE} \cdot \sin\theta$$

Use the following box to calculate the radius of the orbit, R:

$R =$	km	m

20. Once you have the radius of the orbit you can calculate the velocity of the moon using the equation below; where, v, is the linear velocity, ω, is the angular velocity, T, is the period of the moon (in seconds) and R, is the orbital radius of your moon (in metres).

$$v = \omega \cdot R = \frac{2\pi}{T} R$$

$v =$	ms^{-1}

2.3 Calculate the Mass of Jupiter

Using Kepler's Laws you can calculate the mass of the planet.

Kepler's third law states that the square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.

Considering that the planet moves in a circular orbit with no friction, the gravitational force, F_G, equals the centrifugal force, F_C. Therefore, Kepler's third law can be express as:

$$F_G = F_C \qquad \rightarrow \qquad \frac{GMm}{R^2} = m\, a_c$$

$$as \quad a_c = \frac{v^2}{R} \qquad \rightarrow \qquad \frac{GMm}{R^2} = m \frac{v^2}{R}$$

$$and\ as \quad v = \omega \cdot R = \frac{2\pi}{T} R \qquad \rightarrow \qquad \frac{GM}{4\pi^2} = \frac{R^3}{T^2}$$

Note that, M, is the mass of Jupiter and, m, is the mass of the orbiting moon. For the moon, v, is the linear velocity (in metres per second), R, is the radius of its orbit (in metres), ω, is the angular velocity of the moon (expressed in radians over seconds), T, is the orbital period (in seconds) and, G, is the universal gravitational constant, with a value of $G = 6.674 \cdot 10^{-11}\ m^3\, kg^{-1}\, s^{-2}$

Therefore, the quotation previously mentioned is achieved: $T^2 \propto R^3$

$$\frac{GM}{4\pi^2} = \frac{R^3}{T^2} \qquad \rightarrow \qquad \boxed{M_J = \frac{4\pi^2}{G}\frac{R^3}{T^2}}$$

Use the following box to calculate the mass of Jupiter:

Remember to use scientific notation!

$M_J =$	kg

Additional activity: Predict a transit

Did you know?

A transit occurs when one astronomical object passes in front of another. They are similar to eclipses, the difference is only to do with the apparent size of the two objects. If one object is apparently larger in size than the other it is called a transit, but if their apparent size is similar it is called an eclipse.

For example, on Earth eclipses can be seen every year. A solar eclipse occurs when the Moon is between the Earth and the Sun, It causes several places of the Earth to be dark, even when it is daytime. The same phenomenon happens on other planets in the Solar System and other stars.

Phenomena concept (left), solar eclipse (centre) and Mercury transit (right) Credit: CESAR/NASA

The transits of the Galilean moons across Jupiter can be seen from Earth using ground-based telescopes and satellites.

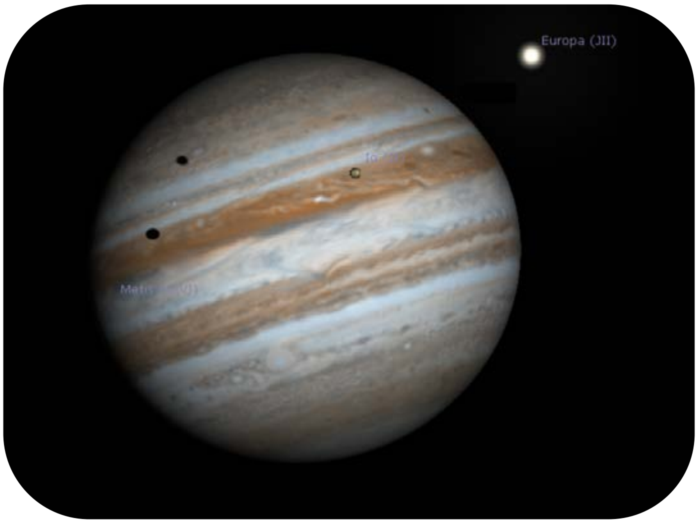

Figure 12: Io and Europa transit, using Stellarium.

Figure 13 shows the trajectory of the moons (X axis) over time (Y axis), this gives a sinusoidal movement, as seen in the figure. But remember, this movement is a straight line as seen from Earth, we saw in Figure 10. In the graph below, the black curve of the graph represents the separation between the moon to Jupiter, given as radii of Jupiter.

Every time the moon is on the “dark part” (called umbra) of the graph it is transiting Jupiter, whether behind it or a background pass. The grey part is assigned to the penumbra.

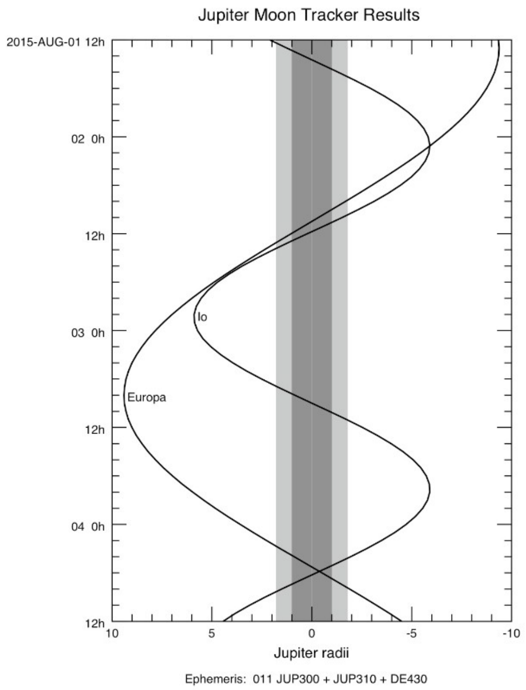

Figure 13: Jupiter Moon Tracker Results

Predict the transit generated by your favourite moon on Jupiter surface:

> Note: If your moon is Callisto you may need to change to another for this activity. A transit also depends on the angle of inclination of the orbit and as Callisto is so far from Jupiter its transit is not visible from the Earth.

- In the case of Io and Europa the date and time of one transit is already in the script. But try to find another, press for moving to your current date and time.
- If you have Ganymede, you will have to look for it. One way you can do this is to move forward in time, for example at x300 time rate, and stop the animation when you find it.

It is recommended to use the same moon as chosen before as favourite as we are going to use some of the already calculated values (T, the period).

1. Open Stellarium.
2. Open a console and add these lines to your previous script.

```
StelMovementMgr.zoomTo(0.0167, 5);
core.setDate("2018:08:17T00:20:50","utc");
core.setTimeRate(300);
```

3. Write down the date (start and end times) of a transit of your chosen as favourite moon.

Note: the format (YYYY-MM-DD hh:mm:ss)

Beginning of the 1st transit	End of the 1st transit

4. Now your job is to predict the next transit. When is the next transit going to be?

Note: Remember from previous activities that the movement of the moons is periodic, and that you have already calculated the period.

Beginning of the 2nd transit	End of the 2nd transit

Do you think this transit will be seen with telescopes on Earth? And with space telescopes? Explain your answers.
