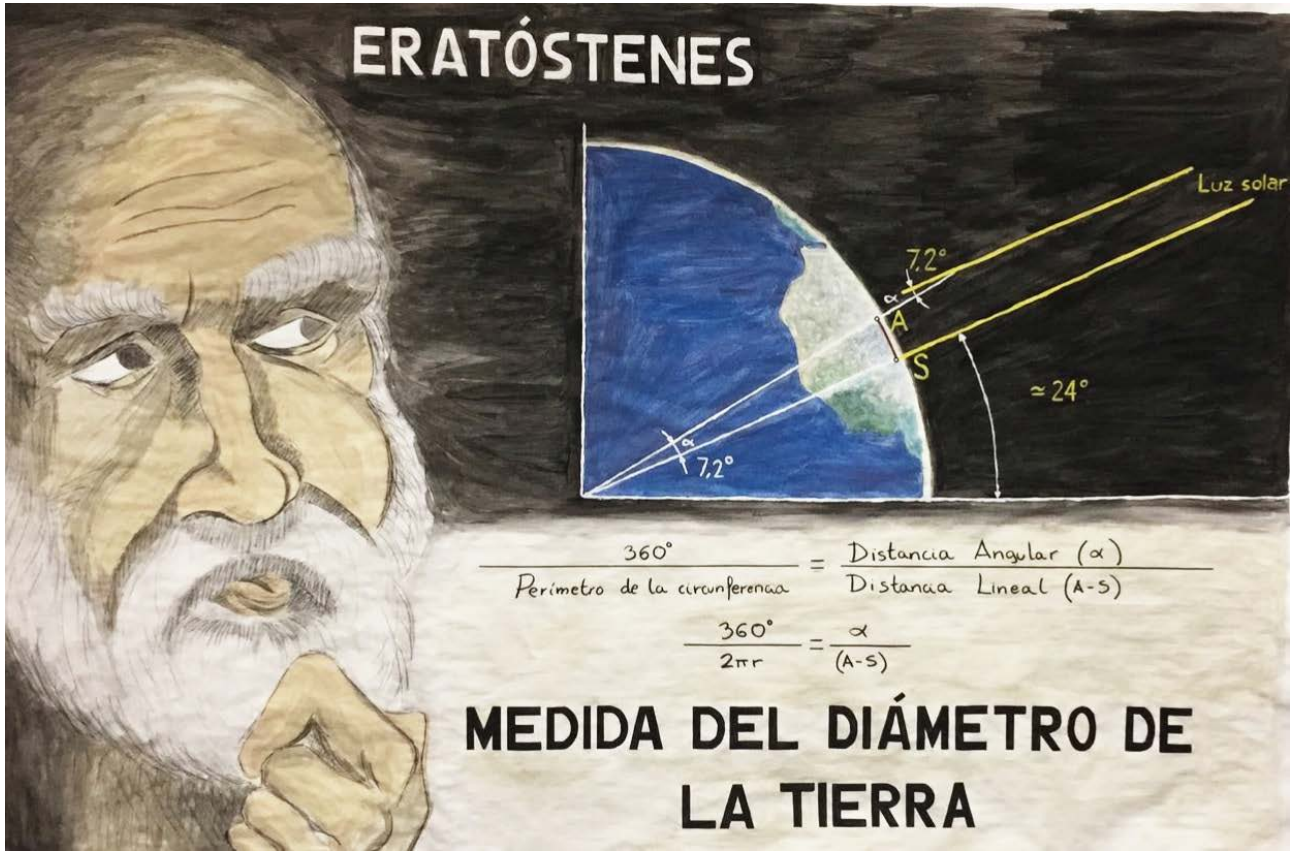
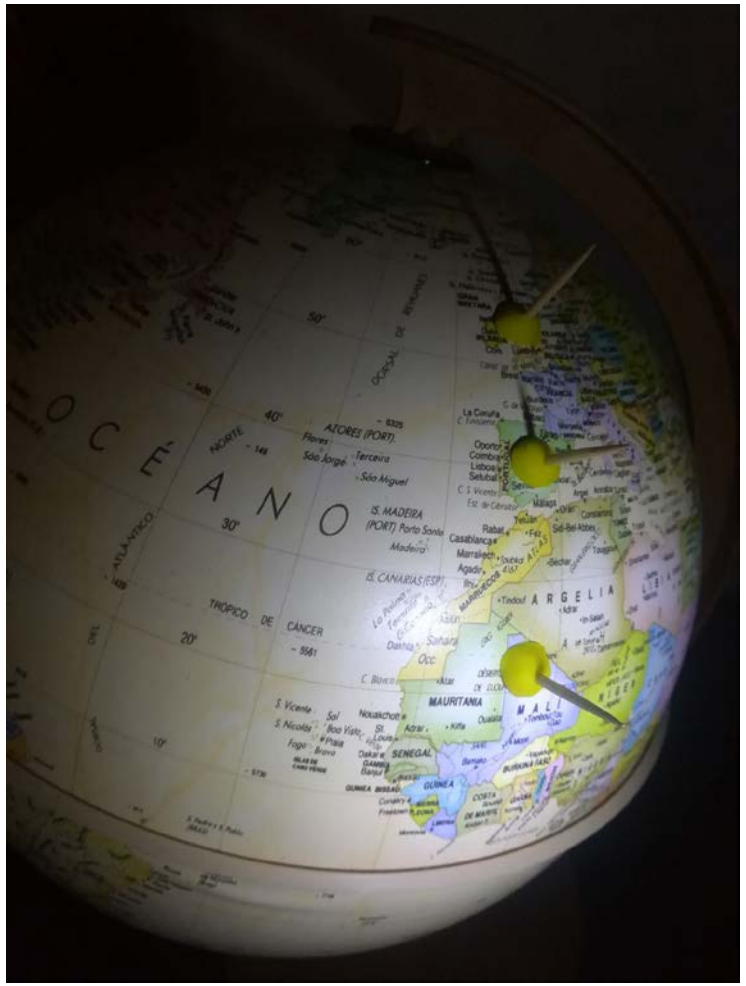
Eratosthenes experiment

Caude 21 Mar 2018





CesaR

esa

ERATOSTHENES EXPERIMENT

Info needed before the experiment

- Get locations geometry and local noon time for the date of experiment (NOAA webpage, https://www.esrl.noaa.gov/gmd/grad/solcalc/)
- Compute ground distances (via Google Earth) between schools and to equator (or the location where shadow is zero, sun is zenithal)
- Get a template/table for the computations and measurements records
- Get the EA lesson plans (online, web, http://eratosthenes.ea.gr/content/lesson-plans)
- Set up Videocon between the schools (Google hangout)
- Material: the gnomon, chalk/pen, paper for measurements, bubble balance, clock (mobile phone), rope, angle measurement device, Earth ball, toothpicks
- Make sure the schools are ready on their side, and have talked to each other and arranged for a date
- Camera for taking pictures for the web news

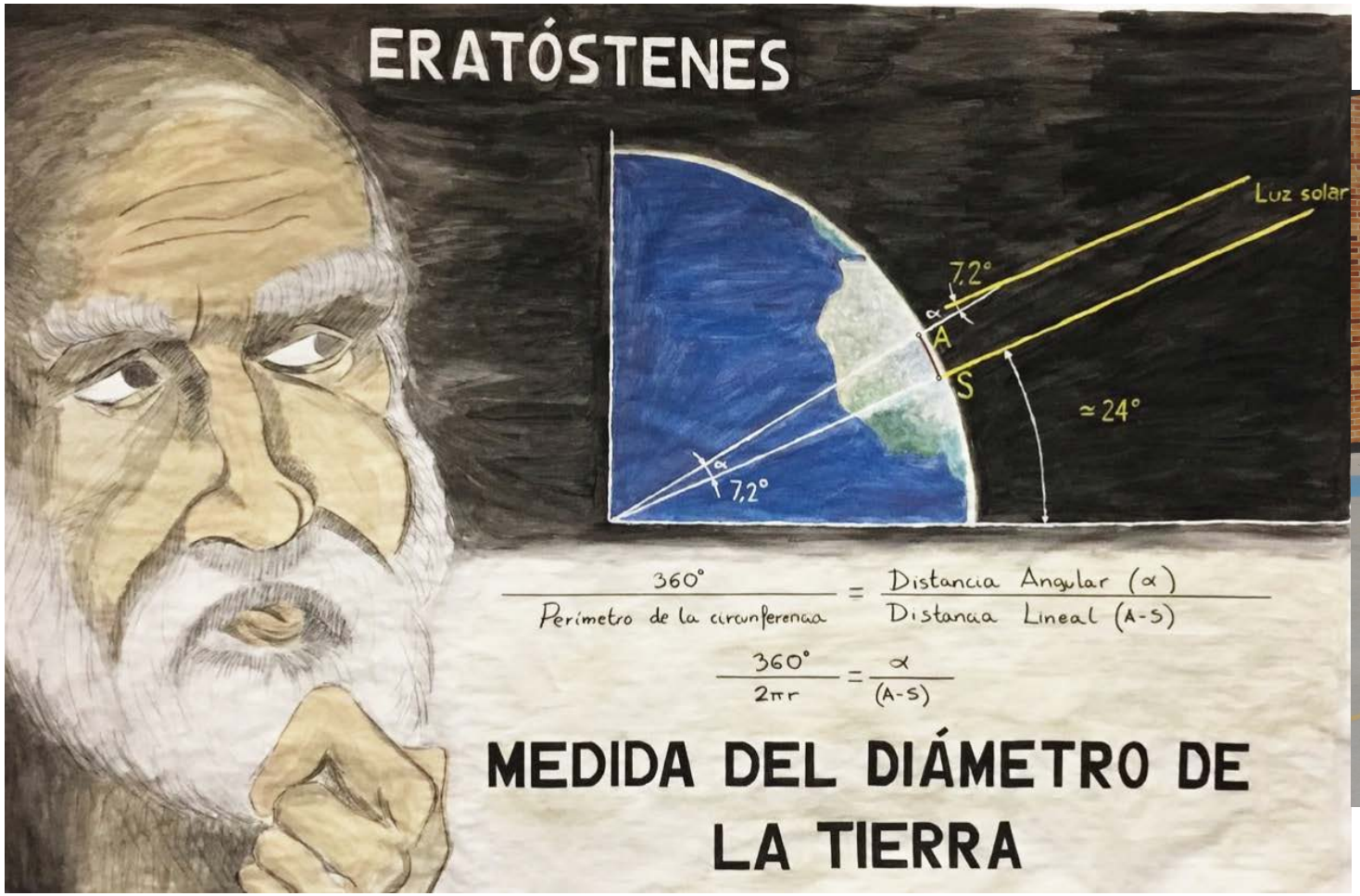
Tasks during the experiment day

- Give a small intro to the experiment (in the class, or outside)
  - Noon-1h
- Follow the measurements with the kids, while explaining he experiment, geometry, how to measure etc…
  - Noon -30min to Noon +30min
- Keep the hangout connection, make sure it works, talk when necessary, give the word to students
- Wrap-up the activity
  - Noon +30min, explanation of results if available, especially from the two locations. Wrap up words.

Tasks after the experiment

- Get measurements from both schools and derive the Earth radius
  - Based on the school results
- Analyze the measurement errors (shadow, inclinations, outliers)
- Complete a science news for the CESAR web
- Announce results to EA for their own news releases

Pictures 21 March 2018



Pictures 21 March 2018

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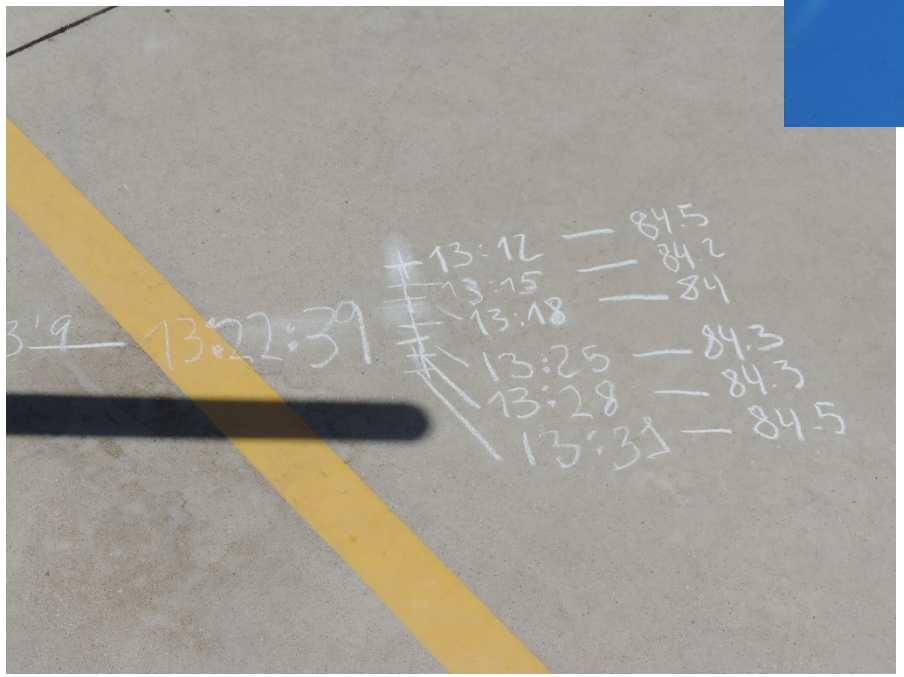
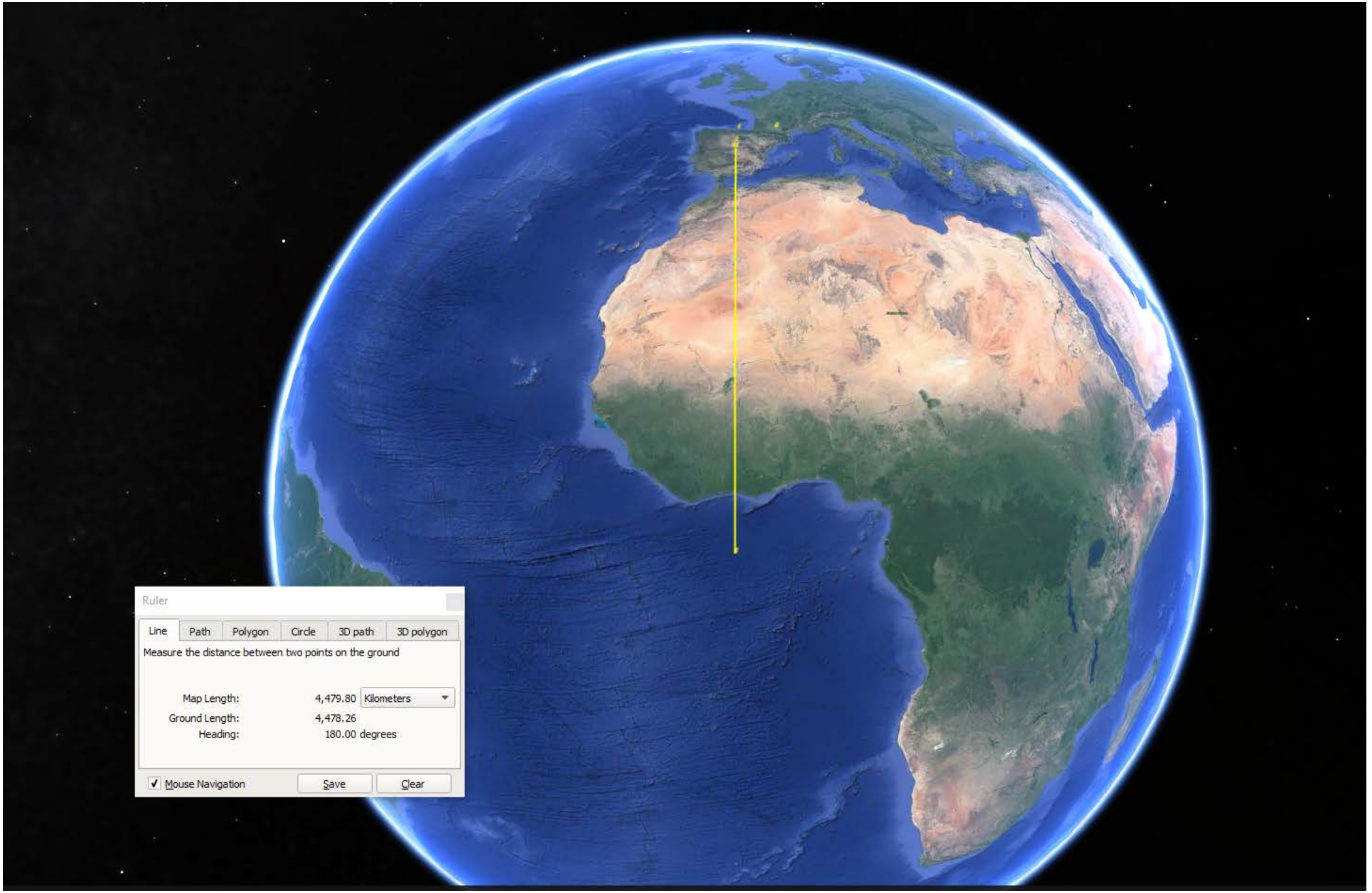


Pictures 21 March 2018

80 students, 1, 2 ESO

Locations geometry

- Coordinates
  - Caude 40°27'33.46" N --- 3°52'31.28" W
  - Granada 37°10'36.62"N --- 3°35'54.30"W
- Distances
  - Distance Caude – Equator 4479km
  - Distance Granada – Equator 4116km
  - Distance Caude – Granada 365km
- Local noon (21Mar2018)
  - Caude 13:22:39
  - Granada 13:21:33


Ruler
Line
Path
Polygon
Circle
3D path
3D polygon
Measure the distance between two points on the ground
Map Length:
4,479.80
Kilometers
Ground Length:
4,478.26
Heading:
180.00 degrees
Mouse Navigation
Save
Clear


Roman amphitheater
Japanese Garden
Hotel Royal Wilson
Casa Horreo de Hazas
Cantabrian Mountains
Pyrenees Mountains
Cantalejo camping
Barcelona
CESAR Robledo Optical telescope
CESAR Cebreros Optical Telescope
Home
Balearic Sea
Mallorca
Eivissa (Ibiza)
Lisbon
Sierra Morena Mountains
Eratosthenes Granada
Gulf of Cádiz
Algiers
Alboran Sea
Ruler
Line
Path
Polygon
Circle
3D path
3D polygon
Measure the distance between two points on the ground
Map Length:
365.15
Kilometers
Ground Length:
365.13
Heading:
356.33 degrees
Mouse Navigation
Save
Clear

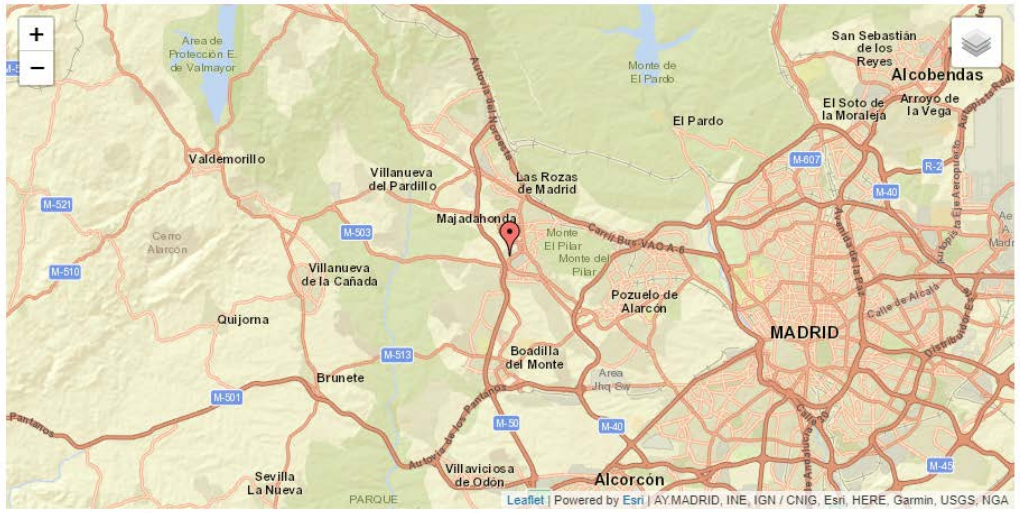
Noon local times

NOAA Solar Calculator

Find Sunrise, Sunset, Solar Noon and Solar Position for Any Place on Earth

Show: World Cities | U.S. Cities | GMD Observ.'s | GMD Data Sites | SurfRad

Click one of the small pins near (and in the same time zone as) your desired location. Use the control on the left side of the map to zoom in or out. Place the large pin in the exact desired location. You can use the Save button to have your computer remember the current location for next time. Check the DST check box if Daylight Saving Time is in effect for your site.



Location:

Latitude:	Longitude:	Time Zone:
40.459201	-3.8755238	1

DST?

Save Location

Date:

Day:	Month:	Year:
21	Mar	2018

Local Time: 11 : 16 : 19 PM Use Current Time

Result

Equation of Time (minutes):	Solar Declination (in°):	Apparent Sunrise (hh:mm):	Solar Noon (hh:mm:ss):	Apparent Sunset (hh:mm):	Az/El (in °) at Local Time:
-7.18	0.3	07:18	13:22:39	19:28	136.33 / 40.67

Show Sunrise | Show Sunset | Show Azimuth

NOAA Solar Calculator

Find Sunrise, Sunset, Solar Noon and Solar Position for Any Place on Earth

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Location:

Latitude:	Longitude:	Time Zone:
37.175637	-3.5989665	1

DST?

Save Location

Date:

Day:	Month:	Year:
21	Mar	2018

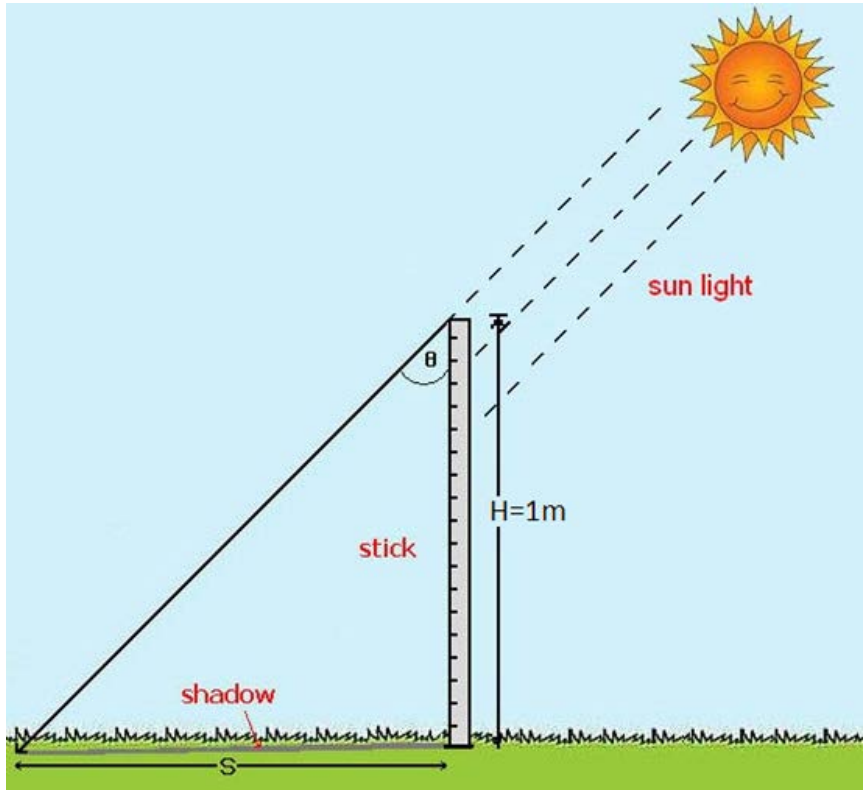
Local Time: 11 : 14 : 07 PM Use Current Time

Result

Equation of Time (minutes):	Solar Declination (in°):	Apparent Sunrise (hh:mm):	Solar Noon (hh:mm:ss):	Apparent Sunset (hh:mm):	Az/El (in °) at Local Time:
-7.18	0.3	07:17	13:21:33	19:27	133.96 / 42.85

Show Sunrise | Show Sunset | Show Azimuth

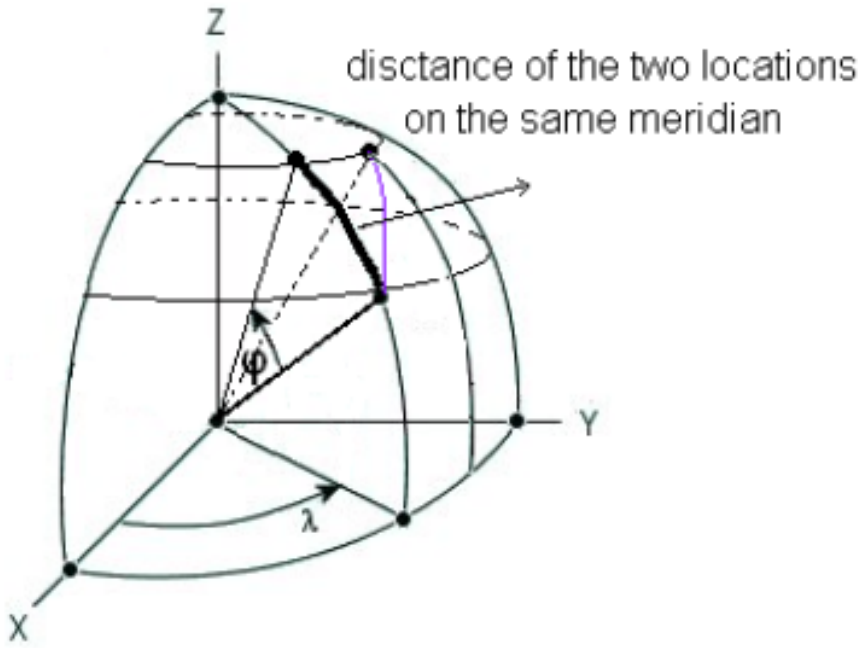
Shadow and incidence angle



- Shadow length
  - S = H * tg(Θ)
- Θ = Latitude, in equinox (20 Mar 2018)

$$tan\theta = \frac{S}{H} \rightarrow \theta = arctan\left(\frac{S}{H}\right)$$

Earth radius



$$arc = radius \times angle_{rad}$$

- Method 1
  - $Radius = \frac{arc}{angle_{rad}} = \frac{distance_{(cau-gra)}}{(\Theta_{cau} - \Theta_{gra})_{rad}} = \frac{distance_{(cau-gra)}}{(\Theta_{cau} - \Theta_{gra})_{deg} \times \frac{2 \times pi}{360}}$
- Method 2
  - $\frac{distance\ cau-gra}{Earth\ circunference} = \frac{\Theta cau - \Theta gra}{360}$

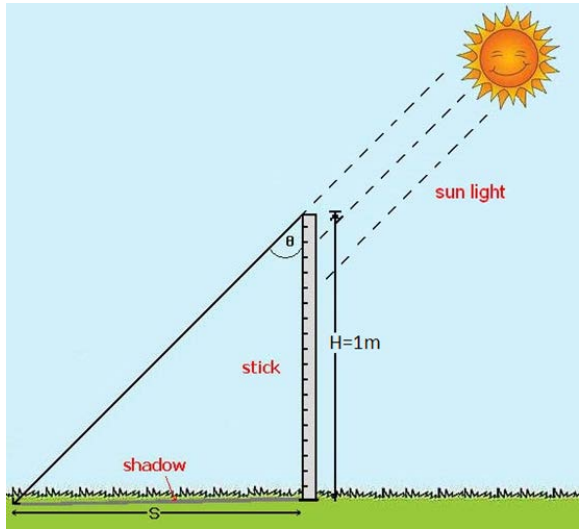
Theoretical results for reference

- Shadow length Caude
  - For H=1m and Θ = 40.45° $(latitude)$
  - S = H*tg(Θ_{cau}) = **85.25cm**
- Shadow length Granada
  - For H=1.01m and Θ = 37.17° (latitude)
  - S = H*tg(Θ_{gra}) = **76.57cm**

$$Radius = \frac{distance_{(cau-gra)}}{\left(\Theta_{cau} - \Theta_{gra}\right)_{rad}} = \frac{distance_{(cau-gra)}}{\left(\Theta_{cau} - \Theta_{gra}\right)_{deg} \times {}^{2 \times pi}/_{360}}$$

$$Radius = \frac{365\ km}{(40.45-37.17)\times {}^{2\times pi}/_{360}} = \frac{365\ km}{(3.28)\times {}^{2\times pi}/_{360}} = 6376\ \text{km}$$

Measurements (shadow length, S)



CAUDE, MADRID

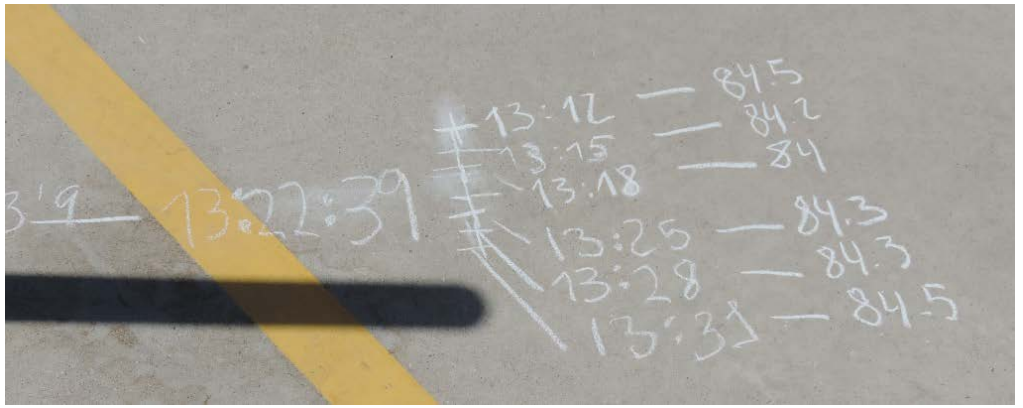
Theory: 85.25cm

- Measurement 1
  - Length_1 = 86 cm
  - Angle_1 = 40deg
- Measurement 2
  - Length_2 = 83.9 cm
  - Angle_2 = 40 deg

GRANADA

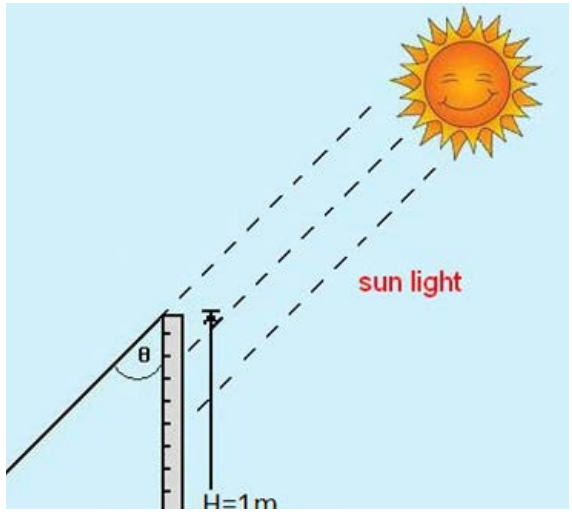
Theory: 76.57cm

- Measurement 1
  - Length_1 = 76.5 cm



Result (angle Θ, based on S and H)

$$\Theta = \operatorname{atan}\left(\frac{S}{H}\right)$$



CAUDE, MADRID

Theory Θ = 40.45°

- Length_1 = 86 cm

$\Theta cau = \operatorname{atan}(\frac{86\ cm}{100\ cm})$ = 40.6955°

- Length_2 = 83.9 cm

$\Theta cau = \operatorname{atan}(\frac{83.9\ cm}{100\ cm})$ = 39.9966°

GRANADA

Theory Θ = 37.17°

- Length_1 = 76.5 cm

$\Theta gra = \operatorname{atan}(\frac{76.5\ cm}{101\ cm})$ = 37.1412°

Result (Earth radius, Caude - Granada)

- $Radius = \frac{distance_{(cau-gra)}}{(\theta_{cau}-\theta_{gra})_{rad}} = \frac{distance_{(cau-gra)}}{(\theta_{cau}-\theta_{gra})_{deg} \times {}^{2\times pi}/_{360}}$

  - $Radius1 = \frac{365\ km}{(40.6955-37.1412)\times {}^{2\times pi}/_{360}} = \frac{365\ km}{(3.5543)\times {}^{2\times pi}/_{360}} =$ 5883 km

    488km off, 7.6% ERROR (wrt 6371km mean radius)

  - $Radius2 = \frac{365\ km}{(39.9966-37.1412)\times {}^{2\times pi}/_{360}} = \frac{365\ km}{(2.8554)\times {}^{2\times pi}/_{360}} =$ 7324 km

    953km off, 14.9% ERROR (wrt 6371km mean radius)

Result (Earth Radius, Caude measurements wrt Equator)

$$\Theta = \operatorname{atan}\left(\frac{S}{H}\right)$$

$$Radius = \frac{d_{cau} - d_{Eq}}{(\Theta_{cau} - \Theta_{Eq}) \times {}^{2 \times pi}/_{360}}$$

- Measurement 1
  - Length_1 = 86 cm
  - Angle_1 = 40deg

$\Theta cau = \operatorname{atan}(\frac{86\ cm}{100\ cm})$ = 40.6955°

$Radius = \frac{4479\ km}{(40.6955° - 0) \times {}^{2 \times pi}/_{360°}} =$ 6306 km

65km off, 1.0% ERROR (wrt 6371km mean radius)

- Measurement 2
  - Length_2 = 83.9 cm
  - Angle_2 = 40 deg

$\Theta cau = \operatorname{atan}(\frac{83.9\ cm}{100\ cm})$ = 39.9966°

$Radius = \frac{4479\ km}{(39.9966° - 0) \times {}^{2 \times pi}/_{360°}} =$ 6416 km

45km off, 0.7% ERROR (wrt 6371km mean radius)

Result (Earth Radius, Granada measurements wrt Equator)

$$\Theta = \operatorname{atan}\left(\frac{S}{H}\right)$$

$$Radius = \frac{d_{cau} - d_{Eq}}{(\Theta_{cau} - \Theta_{Eq}) \times {}^{2 \times pi}/_{360}}$$

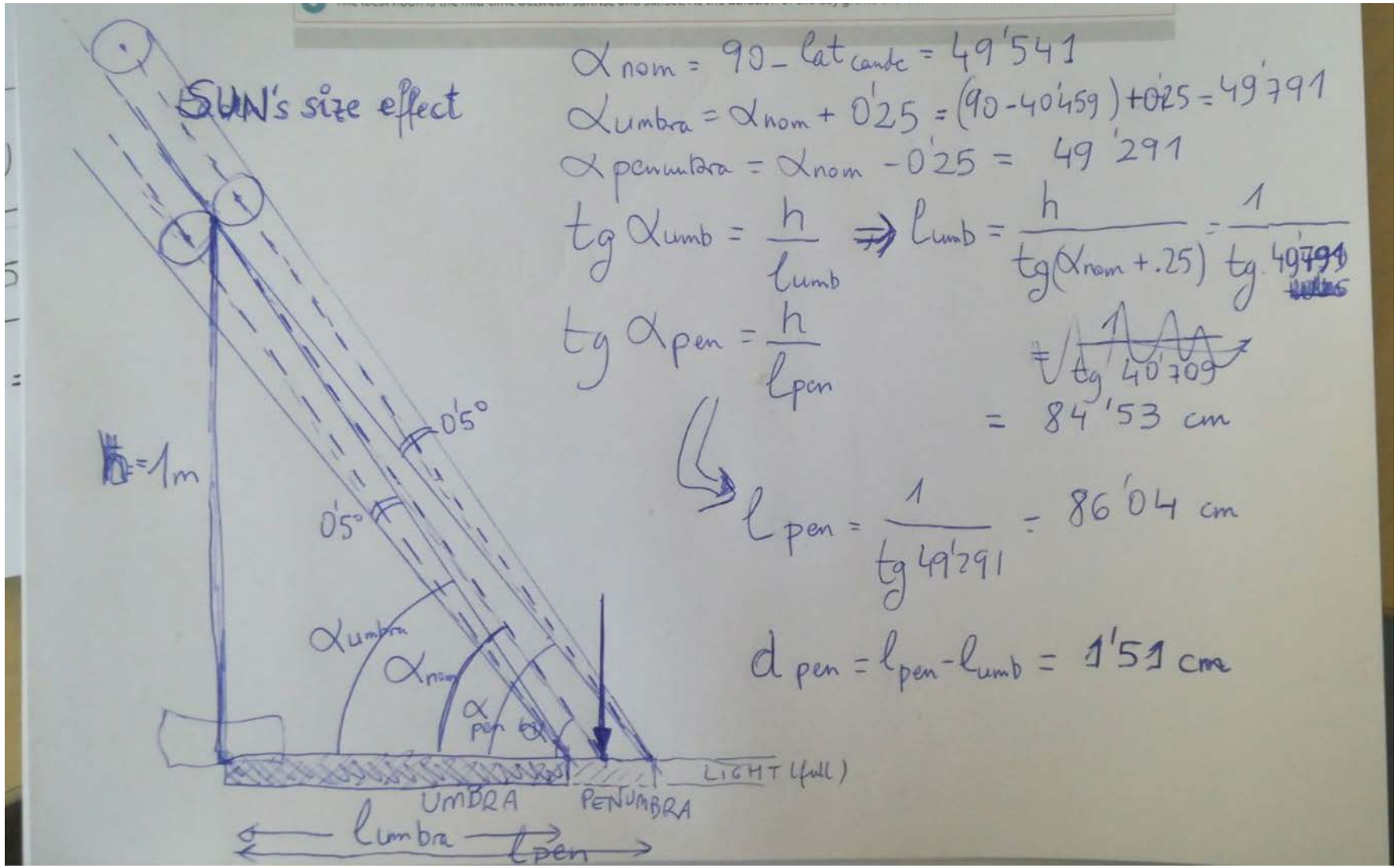
- Measurement 1
  - Length_1 = 76.5 cm

$\Theta gra = \operatorname{atan}(\frac{76.5\ cm}{101\ cm})$ = 37.1412°

$Radius = \frac{4116\ km}{(37.1412° - 0) \times {}^{2 \times pi}/_{360°}} =$ 6350 km

21km off, 0.3% ERROR (wrt 6371km mean radius)

Measurement error due to Sun size



Suggestions for future

(from lower to bigger effort)

- Create a bigger shadow arc (from Noon -1h to Noon +1h, at 3 min intervals)
- Repeat other times of the year (achieve several arcs, need permanent marker or re-use paper)
  - Understand Earth's tilt effect- connect to seasons
  - Understand why local noon changes during the year (analemma, equation of time, ellipticity of orbit)
- Record an analemma picture during the year (either sky pic, or ground)
- Construct a clock
- Construct a meridian / analemma
- Construct a full calendar