Teacher's guide

CESAR Science Case – Rotation period of the Sun and the sunspot activity

The students can use different ways during the laboratory and there are diverse methods to use.

- **Material that is necessary during the laboratory**

  - CESAR Astronomical word list
  - CESAR Booklet
  - CESAR Formula sheet
  - CESAR Student's guide
  - CESAR images of the Sun
  - Matlab, Octave (optional)
  - The software for this Science Case
  - Paper, pencil, ruler, calculator, protractor

Introduction

The tasks that the students need to complete are the following: The first one is to get a set of 3-7 images of the Sun separated by 24h (at least) in the time of acquisition. The second task for the students is to calculate the heliographic coordinates of any of the sunspots. Aligning these images with a solar grid is the easiest way but not the most precise. Using these coordinates, they will be able to estimate the rotation period of the Sun. An extra task of this case is to estimate the size of a sunspot and compare it with the size of the Earth. To finish with, and to see if they understand the sunspot activity, they should be able to estimate where we are in the 11-years solar cycle and try to predict when the next maximum and minimum activity will occur. This is possible by plotting the given data, either by hand or software.

For the first task, there are two possible solutions. If the robotic CESAR solar telescope is available for observation and image capturing, you can download real-time images from the CESAR website. If the CESAR solar telescope is not available or if you don't want to use it, the other option is to download a set of preselected images from the CESAR website for this Science Case and use it. Obviously part of the educational purpose is to approach professional telescopes to the students so they could learn how professional observatories actually works. Moreover they would be able to see how remotely a telescope moves as they want (pointing the Sun in this Science Case). It's not necessary to get a whole set of 7 images, so this Science Case could be done by using two (preferable three) images separated three hours.

It could happen that the day of observation no sunspots are visible. In that case, it's impossible to do this laboratory. For this reason you could check if there are sunspots available before at http://soho.nascom.nasa.gov/sunspots/

At this point it is supposed that you already have the set of images. First, start by handing out all the material. Give them the booklet in advance so that they are well prepared. It's very important for them to understand what sunspots are, how do they change and how do they move during the 11-years cycle.

At the CESAR website for this Science Case, you have all the software that is needed for the laboratory. However, the free software named GIMP is another good choice for the tasks that may require image processing. It is downloadable at http://gimp.com. To help with the plotting of the 11-years solar cycle we provide software too, but you could instead use software that allows you to plot the graph.

Sunspots tracking

Choosing the sunspots from the set of images

As mentioned in the laboratory description, your students need to get the images and its dates for the observations. They need to name some of the sunspots to keep them on track. What name it does not matter, but each spot should have its individual name. Furthermore it's important to choose those spots that are closer to the centre of the Sun. It is preferable because it's easier to appreciate the displacement of a sunspot on the images if it is near to the centre and consequently it's easier to do the measures.

After taking the images with the CESAR telescope, decide if it's worth to process the images or not. It should not be necessary since the quality probably will be good enough. The sunspots must be clearly visible and not blurred. If you consider that sunspots are not clearly visible, try to increase the contrast with an image processing program in order to distinguish them.

A very important thing that has to be mentioned is that the images have to be aligned in E-W direction in the solar disk. This is the first requirement to determinate the orientation of the Sun and its heliographic coordinates. The CESAR images will already been aligned but if you have considered to do this Science Case using your own telescope with an equatorial mount a trick to get the E-W direction is to take pictures while moving in Right-Ascension. If you take two pictures for example and superpose them into a single one, draw a line that connect the same sunspot of the two frames. That is the E-W direction, and you can now rotate the image to see the E-W direction parallel to the mark frame as showed below.

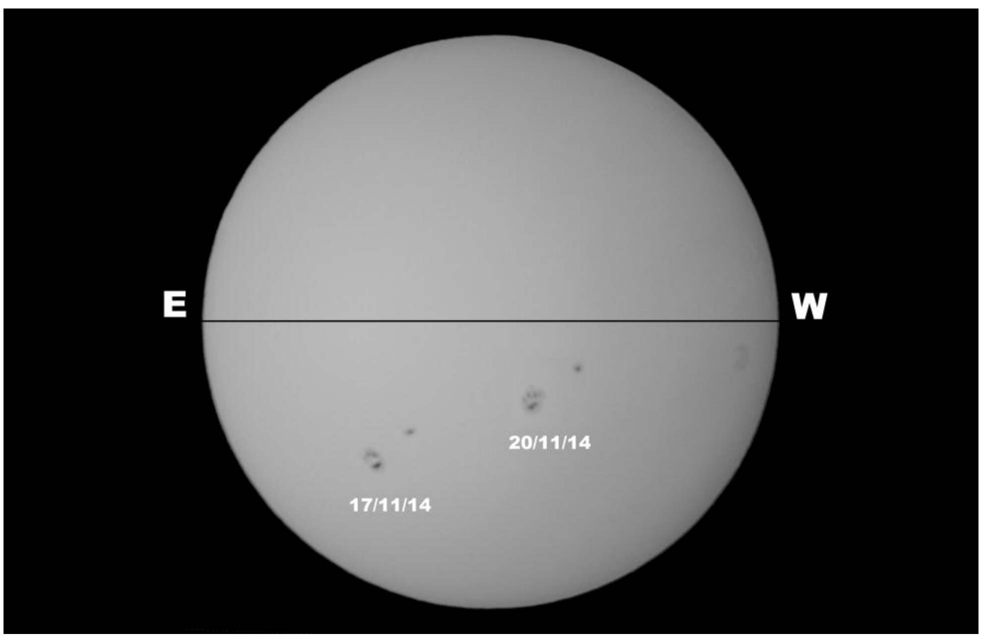


Figure 1: Image showing the *E-W direction where two pictures taken at two separated dates where superposed.* *Credit: CESAR*

Until now, we have a set of images captured at different times and oriented on E-W direction. Now we are going to determinate the heliographic coordinates for the sunspots.

Heliographic coordinates

Determine the heliographic coordinates of the sunspots

The heliographic coordinates are the coordinates of the Sun. As well as a place in the Earth has a longitude and a latitude in the Sun is exactly the same. A sunspot has a longitude and latitude on its surface too. Its latitude is divided into 90 degrees north to the equator, and 90 degrees south to the equator. Its longitude is divided into 360 degrees to the West in the direction of the solar rotation. The Sun as the Earth has a place from which to start counting the longitude degrees.

As well as the Earth has a rotational axis, the Sun too. It's well known that the Earth' rotational axis is tilted 23,5 degrees while the Sun's rotational axis is tilted 7 degrees. If both Earth and Sun rotational axis were parallel between them and perpendicular to the ecliptic, then the E-W direction that we mentioned before would be the equator of the Sun.

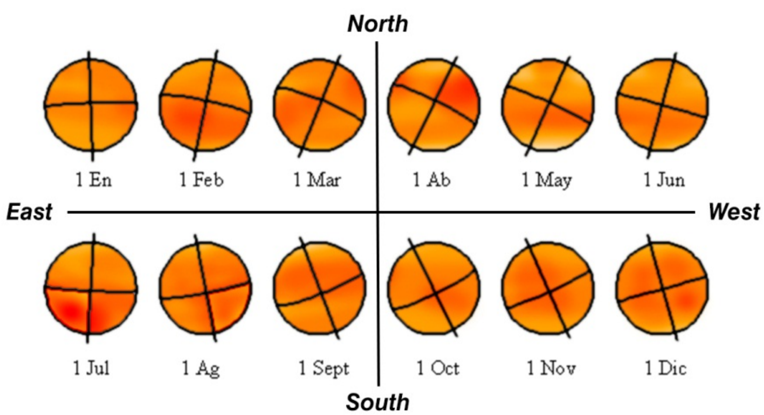


Figure 2: *Inclination of the solar axis along a year viewed from Earth.* *Credit CESAR*

What we are going to do now is to calculate the heliographic coordinates of the sunspots using the coordinates of the sunspots in the E-W and N-S coordinate system. In order to do this, students first have to measure the position of the sunspots from the center of the Sun. The center of the Sun has to be located because is the center of our reference system. Students have to create a table like this:

Sunspot	X coordinate	Y coordinate	R_m	R
1	356 pixels	232 pixels	425 pixels	3115 pixels
…	…	…	…	

Where R_m is $\sqrt{X^2 + Y^2}$ and R is the diameter of the Sun that they have to measure too (only once).

We need now to calculate other values using the X and Y coordinates of the sunspots. These values are the position angle of the sunspot measured from N-S direction (that we call P_m) and the angle between the sunspots with the visual (that we call ρ). Their formulas to be calculated are:

$$P_m = arc\ tg(^X/_Y) \quad (1) \quad \text{and} \quad \rho = arc\ sin\ (R_m/R) - \alpha/2(R_m/R) \quad (2)$$

Where α is de angular diameter of the Sun (on degrees), that you could get on internet or via software (the free software Stellarium is a good choice) for the day of the observation. You could also use the value of 0.5244˚ for summer, 0.5422˚ for winter and 0.5333 for autumn/spring in the North hemisphere.

As well as in the first table, the students have to note the values of P_m and ρ for each sunspot in a table:

Sunspot	P_m(˚)	ρ (˚)
1	56,9	7,8
…	…	…

As we said, the Sun's axis is tilt so we see different angles with the N-S direction along a year. Furthermore the center of the heliographic coordinate system is not the place where E-W and N-S lines cross each other. For all of this, we need to take some values which are:

P: Is the position angle of the solar rotation axis measured from the N-S direction. Positive when it's to the East and negative when it's to the West.

B_0: Heliographic latitude of the solar disk center.

L_0: Heliographic longitude of the solar disk center.

To make this Science Case easier, we could take these values from the Ephemeris. There are some websites such as:

http://alpo-astronomy.org/solarblog/wp-content/uploads/ephems/solar_ephemeris_2016.pdf
http://bass2000.obspm.fr/ephem.php

Here there is a picture showing all these values.

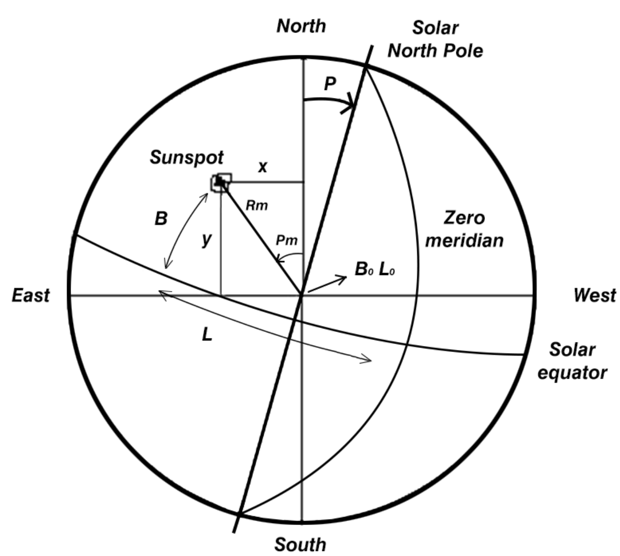


Figure 3: *Picture showing all mentioned values. Credit: CESAR*

At this point, the students have to be able to calculate the heliographic coordinates of any of the sunspots. It would be great if the students calculate the coordinates for at least two sunspots. For this here there are the equations:

$$sen(B) = sen(B_0)\,cos(\rho) + cos(B_0)\,sen(\rho)\,cos(P - P_m) \qquad (3)$$

$$sen(L - L_0) = sen(P - P_m) sen(\rho) cos\,(B) \qquad (4)$$

Where B is the latitude and L the longitude. Note that all of the values that have to be used in the formula are expressed in degrees as well as the results.

To sum up, the most important part is to write down all the data on a paper. The students should create a final table as the one below:

Date/time	Sunspot	Longitude(°)	Latitude(°)	Observations
17/11/14 – 12:00	1	242,813	-18,209	
…	…	…	…	

This is important: as some of the formulas are a bit difficult depending on the knowledge of the students, the software at the website simply ask the students to do some measurements and then automatically calculate the heliographic coordinates.

Grid alignment

Align the images with a grid of heliographic coordinates

Now it's possible to make some interesting calculations but before that we are going to use a grid that will be essential if we want to estimate the heliographic coordinates of a sunspot without doing all the measurements that we explained before.

The first step for this is to download the exact grid for the day when the images were taken. Here is the website for this: http://bass2000.obspm.fr/ephem.php At the website you just have to select the date and time and select "grid" to compute.

Once it has been downloaded we need to superpose it to the image. There are two ways to do this, the first one is to print both (the image and the grid) and draw the sunspot over the grid. The other option is to superpose the grid using software. With this Science Case we provide software to do that. In both cases, is essential to rotate the image the angle P to align the image with the north solar pole.

Now it's very easy to estimate the position of any of the sunspots. We are also able to determinate the size (in terms of heliographic coordinates) of a filament (if for example a h-alpha filter were used) or the size of a group of sunspots as showed in the Science Case: Sun's differential rotation.

The picture result of superposing the image and the grid should be similar to the one below:

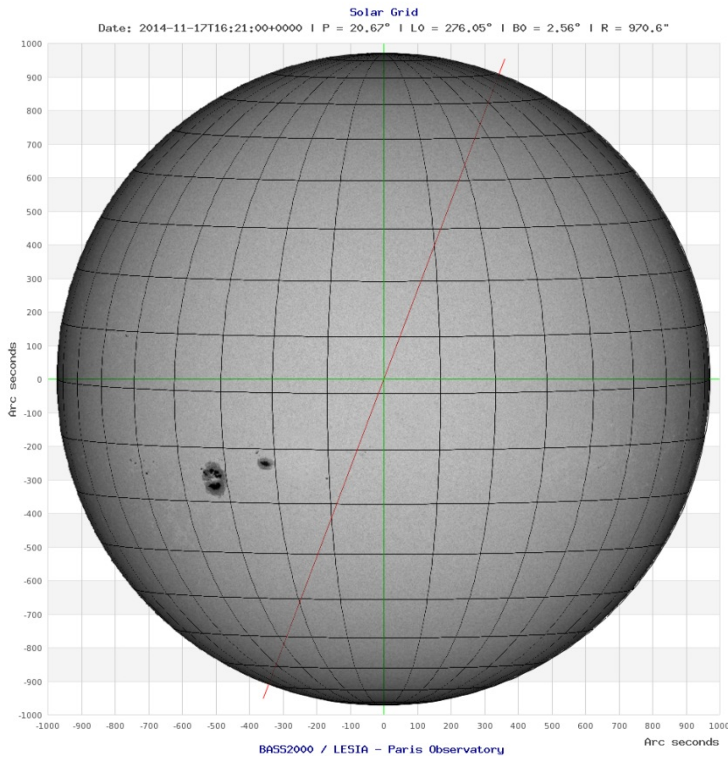


Figure 4: *Heliographic coordinates superposed to an image of the Sun. Credit: CESAR/LEISA*

Rotation period

Calculate the Sun's rotation period using sunspots

To determinate the Sun's rotation period two images at different times are required. The images have to be at least three or more hours separated one from the other. In fact the best idea is to use two images from two different days to do the measurements easier. As an example, the figure 1 shows two pictures of the sunspots three days spaced. That would be the ideal.

Now two different ways to calculate the rotation period are going to be presented. The first one requires some knowledge about sidereal period whereas the other requires a good understanding of trigonometric. In both cases they need to write down the heliographic latitude of the same sunspot for both images. The latitude is not required this time but the students have to repeat the calculations mentioned before to get the longitude. So the first task for the students is to fill this table:

Date/time	Longitude (°)	ΔL(°)

Where ΔL is the difference of longitudes of the sunspot in degrees.

You may consider using more images to track the sunspot. In that case, students can take the mean of the results to ensure a good result.

The next step for the students is to understand the difference between the sidereal rotation period (that we call P) and the synodic rotation period (that we call S). The first one is the time that a sunspot needs to complete a 360 degrees rotation whereas the second one is the time that a sunspot needs to rotate to the same apparent position viewed from Earth.

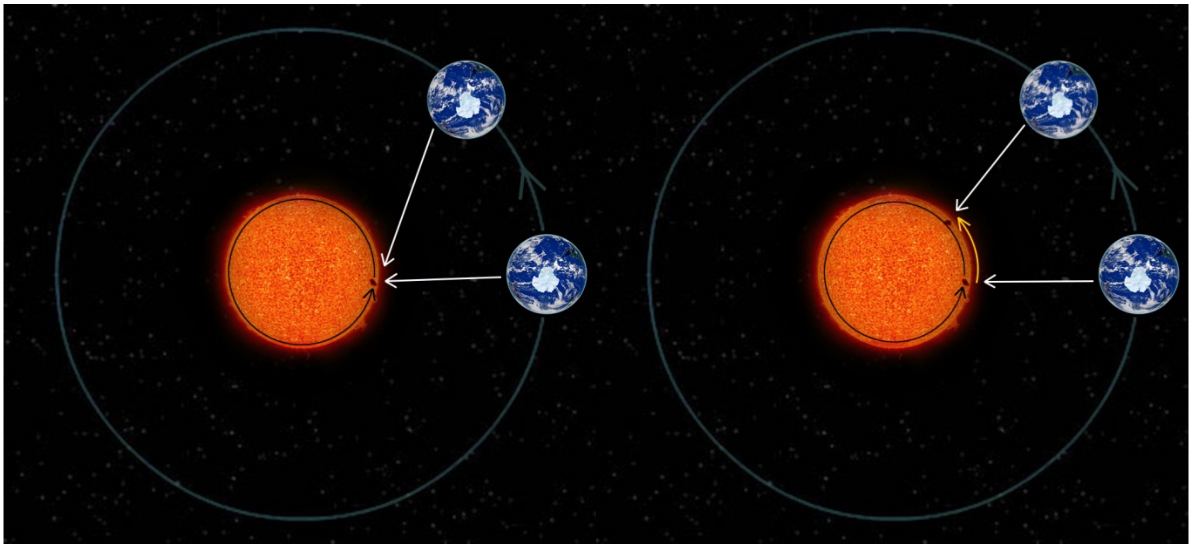


Figure 5: *On the left, the representation of a sidereal rotation of a sunspot. On the right, the representation of a synodic rotation which takes more time to be completed (indicated by the yellow arrow. Credit: CESAR*

To calculate the synodic rotation period the students had to realize that it's as easy as calculate the rate between time and degrees. This is achieved by dividing the time between the two images (Δt) with the change of longitude (ΔL). Then they have to calculate how much time it takes to complete a 360˚ rotation. The equation that the students need to use to calculate S is then:

$$S = 360^\circ \frac{\Delta t}{\Delta L} \quad (5)$$

The value of S should be expressed in days, hours, minutes and seconds, and it should be close to the exact value of 26 days and 14 hours.

Finally, to calculate the sidereal rotation period of the Sun which is the main target of this section, we need to remove the effect of the Earth's translation.

$$P = (S * 365{,}25)/(S + 365.25) \quad (6)$$

This is the real rotation period of the Sun, and the students should get an approximate value of 27d 6h 36'.

The other procedure that we are going to use is based on some trigonometric formulas. A calculation of this rate can be achieved like this (there may be several different ways this is just one of them).

As we have already seen, the movement of a sunspot can be used to obtain the rate of the Sun's rotation. One method to calculate the rotation period of the Sun is to imagine the sunspots seen from the top of the

Sun. By doing this, the angle of solar latitude and longitude can be calculated. Here is an image of how it might look like:

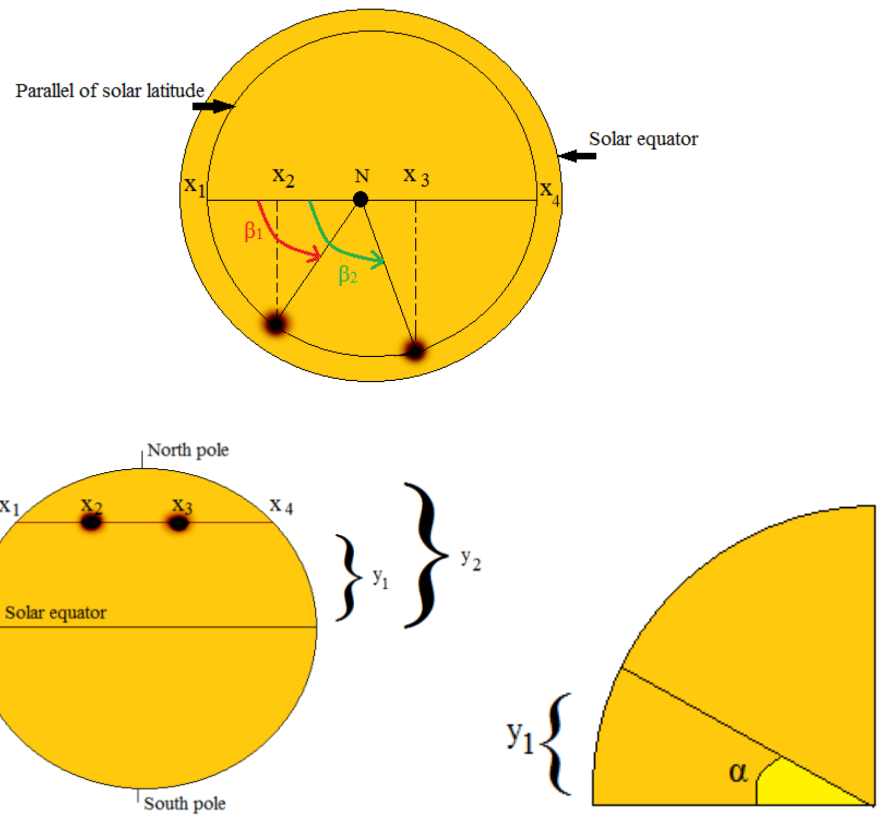


Figure 6: *The upper image is the Sun from above. The bottom left image is the Sun viewed from Earth, and the bottom right image is the Sun from the center.* ***Credit****: CESAR*

In the figure 6, α is the solar latitude, β_1 is the solar longitude angle of the spot at first time while β_2 is the angle of the spot at time two. The first one is measured from the edge of the Sun. As the student may know, the sunspots are always located at the same latitude measured from the equator of the Sun (or from the pole equivalently). Along the line of latitude, the student can identify four locations along that corresponding latitude that are distinguished by their x-coordinates. Here is an example of a coordinate table that the student may come up with and that is helpful.

X_1	The sunspot on the left “edge”
X_2	Sunspot in the first image
X_3	Sunspot on the second image
X_4	The sunspot on the right “edge”

We are now at the point where the β_1 angle needs to be figured out, but also β_2 from the left edge of the second image of the spot. The two angles have a difference:

$$\Delta\beta = \beta 2 - \beta 1 \qquad (7)$$

This is the Sun's angle of rotation between the times of the two images. We first need to find the Sun's semi-diameter (also known as the radius) in pixels (depending on what method they chose) at that latitude. We can call it φ. Using the figure below it is obvious with trigonometric relations that:

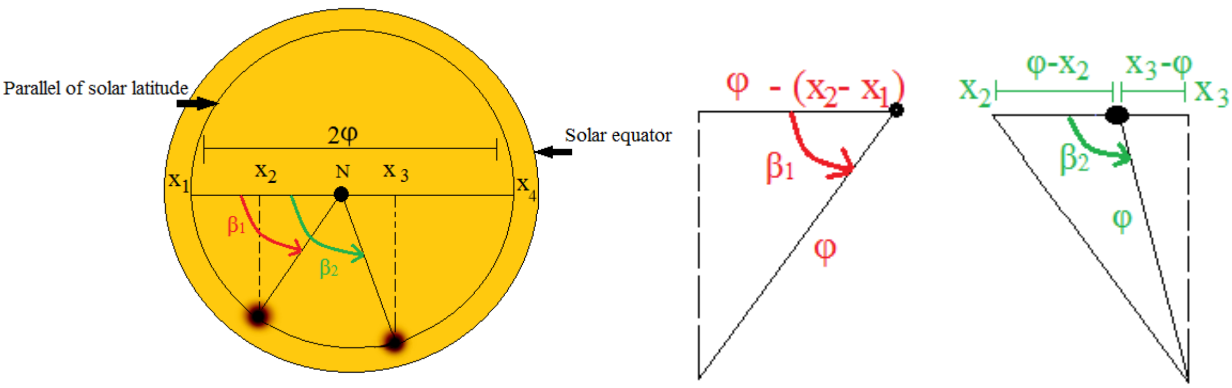


Figure 7: *Trigonometric explanation.* ***Credit:*** *CESAR*

$$\varphi = \frac{x_4 - x_1}{2} \qquad (8)$$

If we take a look at the first image, we can conclude that the angle β_1 satisfies the trigonometric relation:

$$cos(\beta_1) = \frac{1}{\varphi}(\varphi - x_2 + x_1) = \frac{\frac{x_4 - x_1}{2} - x_2 + x_1}{\frac{x_4 - x_1}{2}} \text{ giving: } \beta_1 = cos^{-1}\left(\frac{\varphi - (x_2 - x_1)}{\varphi}\right) \qquad (9)$$

Here, x_1 and x_2 are the coordinates of the edge to the left of the Sun and of the sunspot measured at that same latitude. To get the second angle, β_2, we need a more complicated evaluation. Take a look at the figure. It can be seen from it (by trigonometric relations) that:

$$cos(\pi - \beta_2) = cos\,(\beta_2) = \frac{Adjacent\ side}{hypotenuse} = \frac{x_3 - \varphi}{\varphi} \text{ giving: } \beta_2 = cos^{-1}\left(\frac{x_3 - \varphi}{\varphi}\right) \qquad (10)$$

$$\Delta\beta = \beta 2 - \beta 1 = cos^{-1}\left(\frac{x_3 - \varphi}{\varphi}\right) - cos^{-1}\left(\frac{\varphi - (x_2 - x_1)}{\varphi}\right) \qquad (11)$$

As we saw in the previous method, the sunspot's time of rotation at this height or latitude is the time it would take for it to go around the entire 360° of longitude at the solar latitude. These times can be called t_1 and t_2, and once they are determined you can calculate the time interval $\Delta t = t_2 - t_1$ (in days):

The longitudinal angle's rate of change that is measured in degrees/day is:

$$\frac{\Delta\beta}{\Delta t} = \frac{cos^{-1}\left(\frac{x_3 - \varphi}{\varphi}\right) - cos^{-1}\left(\frac{\varphi - (x_2 - x_1)}{\varphi}\right)}{(t_2 - t_1)} \qquad (12)$$

And the sunspot's rotation period T expressed in days is (note that is similar to the equation five):

$$T = 360^{\circ}\frac{\Delta t}{\Delta \beta} \quad (13)$$

We know that the Sun is a gaseous body (rather than a solid one). Its rate of rotation depends to some degree on its latitude. It actually rotates faster at the equator and more slowly at higher latitudes as it's developed in the "Differential rotation of the Sun and the Chromosphere" Science Case. These are formulas that can be used. Of course, the student is free to choose what method to use.

Size of a sunspot

Get the size of a sunspot or a group of then and then compare it with the Earth size

It was mentioned in the Science Case that it is possible to calculate the sunspots sizes by using the images. Let the students look up the values for the diameter of the Sun and the Earth in kilometres, and the diameter of the Sun and a spot on their images in pixels (this extra task is also included in software for this Science Case where they could measure this sizes on pixels). If they have the same units, they can cross multiply to get the magnitude in size of the sunspot in km. Here is one example:

The size of the sunspot: 21 pixels

The size of the Sun on the drawing: 936 pixels

Average diameter of the Sun: 1.391.000 km

Diameter of Earth: 12.742 km

To measure the size of a sunspot they could measure diameter of the umbra or the diameter of the umbra and the penumbra too, that is larger. They could also measure the size of a group of sunspots. The simple relationship to calculate the value is the following:

$$\begin{aligned} &Measured\ size\ of\ the\ sunspot/Measured\ size\ of\ the\ Sun \\ &\qquad = Real\ size\ of\ sunspot/Real\ size\ of\ the\ Sun \quad (6)\end{aligned}$$

For the example we get:

$$Real\ size\ of\ sunspot = \frac{21 * 1.391.000}{936} = 31.208\ km$$

And finally, how many Earth diameters can fit in a sunspot with the calculated size? The simplest way to compute it is to divide the size of the sunspot by the Earth's diameter

$$Size\ of\ sunspot\ (in\ Earths) = Real\ size\ of\ sunspot/Size\ of\ Earth \quad (7)$$

Numerically:

$$Size\ of\ sunspot\ (in\ Earths) = 31.208/12.742 = \mathbf{2.45}\ Earths!$$

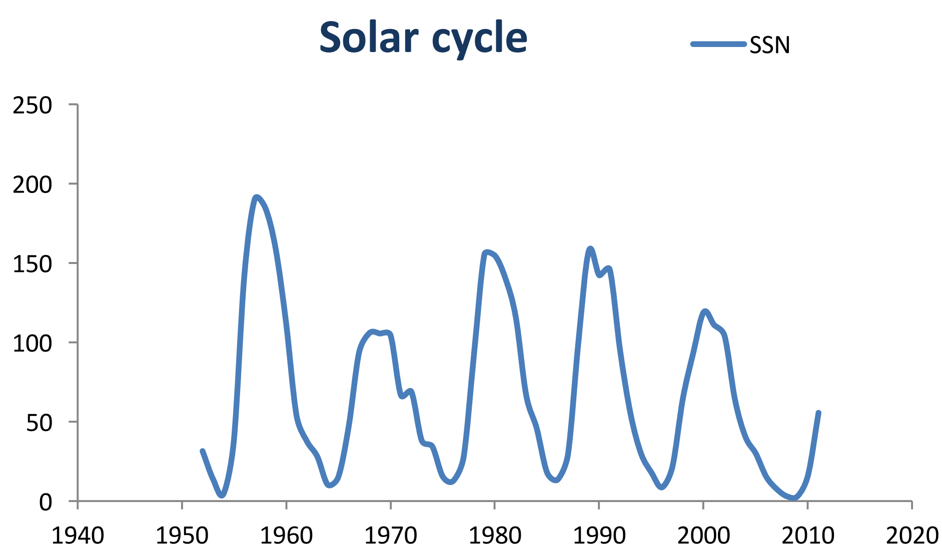
The Earth is approximately 110 times smaller than the Sun so the students have to realize about this.

Solar Cycle

The next exercise is to use the old sunspot data that is provided and let the students plot it to see if they can predict if we are in a solar maximum or minimum or in the middle of both. Of course, the student can plot the collected data on a paper by hand but is preferable using software. It is basically up to them, but you as a teacher you should maybe go through simple Matlab *(or another language)* coding just to get an approach to some programming. Power point is also a good software to use when data needs to be plotted.

The website http://solarscience.msfc.nasa.gov/greenwch/spot_num.txt has data from January 1949 up to now. They can basically choose whatever year they want, or use the examples we provide. If they want to pick different years, they need to calculate the average SSN (Sun Spot Number) for each year. This can be done by adding the SSN for each month in each year, and then divide it by 12 months.

After following these steps, they should get a graph similar to this one below. By looking at it, they should conclude if we are in a maximum or a minimum. It is obvious in the plot that we are on a solar maximum.



There is another method to estimate where we are in the solar cycle. It is based on the Maunder Butterfly Diagram explained in the Booklet for this Science Case. Based on it, what we need to use is the latitude values of the sunspots. With a range between 5 and 35 degrees, the closer a sunspot is to the equator, the closest to the minimum we are. If the cycle takes 11 years on average to be completed, an easy proportion will give the students an idea of where we are on in. Let's take an example: imagine that the observation day there are two sunspots with different latitudes. Then the students have to create a table like this:

Sunspot number	Latitude of the sunspot
1	15˚ N
2	-19˚ S

Either if the sunspot is the north or south, the absolute value is the one that have to be considered. So the students have to take the mean of the latitudes (17 in this case). With a table that shows the variance of the latitude with the percent of completed cycle it easy to check that 17 degrees equals to 60% of the solar cycle.

Sunspot latitude	Cycle's year	Cycle's percept	Estimate date
35	0.00	0.00	Jan 2008 - Mid May 2008
34	0.37	3.33	Mid May 2008 - Sep 2008
33	0.73	6.67	Sep 2008 - Mid Feb 2009
32	1.10	10.00	Mid Feb 2009 - Jun 2009
31	1.47	13.33	Jun 2009 - Mid Nov 2009
30	1.83	16.67	Mid Nov 2009 - Mar 2010
29	2.20	20.00	Mar 2010 - Mid Aug 2010
28	2.57	23.33	Mid Aug 2010 - Dec 2010
27	2.93	26.67	Jan 2011 - Mid May 2011
26	3.30	30.00	Mid May 2011 - Sep 2011
25	3.67	33.33	Sep 2011 - Mid Feb 2012
24	4.03	36.67	Mid Feb 2012 - Jun 2012
23	4.40	40.00	Jun 2012 - Mid Nov 2012
22	4.77	43.33	Mid Nov 2012 - Mar 2013
21	5.13	46.67	Mar 2013 - Mid Aug 2013
20	5.50	50.00	Mid Aug 2013 - Dec 2013
19	5.87	53.33	Jan 2014 - Mid May 2014
18	6.23	56.67	Mid May 2014 - Sep 2014
17	6.60	60.00	Sep 2014 - Mid Feb 2015
16	6.97	63.33	Mid Feb 2015 - Jun 2015
15	7.33	66.67	Jun 2015 - Mid Nov 2015
14	7.70	70.00	Mid Nov 2015 - Mar 2016
13	8.07	73.33	Mar 2016 - Mid Aug 2016
12	8.43	76.67	Mid Aug 2016 - Dec 2016
11	8.80	80.00	Jan 2017 - Mid May 2017
10	9.17	83.33	Mid May 2017 - Sep 2017
9	9.53	86.67	Sep 2017 - Mid Feb 2018
8	9.90	90.00	Mid Feb 2018 - Jun 2018
7	10.27	93.33	Jun 2018 - Mid Nov 2018
6	10.63	96.67	Mid Nov 2018 - Mar 2019
5	11.00	100.00	Mar 2019 - Mid Aug 2019

Be careful because the last column shows an estimation of the date, but it has been considered that we are in an 11 years solar cycle and that's not always true. Some historical cycles took 14 years to be completed while others took just 9 years. For this reason the last column shows if we are close to an 11 years solar cycle or not.

Finally one last method to estimate if we are close to the maximum or minimum is to determinate the Wolf number. The Wolf number is calculated by counting the number of individual spots (s) and the number of sunspot groups (g) and using a factor that depends on the instrumentation that we use (k). For this exercise we are going to take $k = 1$. The formula is the next:

$$W = k(10g + s) \quad (14)$$

To count the number of sunspots and groups you could either use the images and let the students count them on a paper or use the software provided to calculate the Wolf number by clicking the sunspots and groups on the uploaded image.

The Wolf range of values is between 0-10 for minimum activity and 140-170 for the maximum. This method sometimes fails because we can only observe half of the solar disk so it could happen that all of the sunspots appeared in the non-observable face of the Sun. Furthermore the images that we provide cannot be compared with the images of space solar telescopes such as SOHO. For this reason some of the sunspots won't be visible and the estimated Wolf number that the students will give to you will be probably below the official number for that day that you could take from the website: http://www.astrosurf.com/obsolar/wolfnumber.html

When all of these three methods were done, a global idea of where in the solar cycle we are should be known. Have a discussion session with the students and see if they have understood the theory behind solar activities. If the theory is not completely understood, make sure to explain and answer the questions.