The Parallax Effect

CESAR's Booklet

The Parallax Effect is a difference in the apparent position of an object when viewed along two different lines of sight. The Parallax it's measured by the semi-angle of inclination between those two lines.

If this is the first time you read about Parallax, this definition may sound a bit complicated, but Parallax is actually really simple, and you are seeing it every day without noticing. This brief booklet introduces parallax as seen in everyday life, and then, after properly describing it, we'll seek for it's application in astronomy.

Parallax in everyday life

Nothing like an example to clear out a tough definition: Imagine you're in a road trip, lets say from Berlin to Madrid. At some point of the trip, Paris will be at our right, far away from the road, as in Image 1.

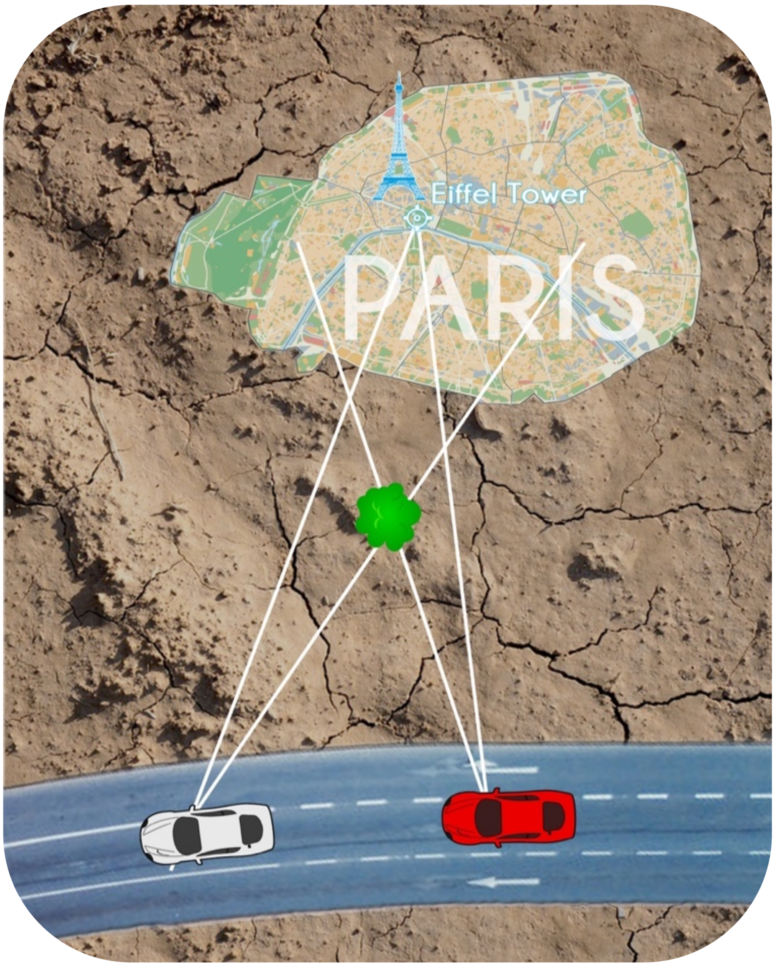

Image 1: Road trip example

You may notice there's a green tree close to the road. If you were driving the red car and you looked at Paris, this tree would seem to be at the left of the Eiffel Tower. But if you were driving the white car and still looking at Paris, then the tree would look as if it was at the right side of the Eiffel Tower. So even if the tree is still, it's seen in two different positions depending on where you are looking from. This is the parallax effect, a difference in the apparent position of an object when viewed along two different lines of sight. If this is not clear to you, try to imagine how would a Paris picture look like if you took it from each car. Try to imagine where would the tree be in each picture.

Parallax Geometry

Now let's move one step further, we'll use the previous example to study a general parallax situation. First of all, the notation, for now on we'll use:

- Brief titleless introduction.
- R to name the observed object (the tree).
- S to name the background (Paris).
- T to name the place from where the object is observed (the road).
- A to name the first observer's exact location (the red car).
- B to name the second observer's exact location (the white car).
- A' to name the place where R appears to be as seen from A (left side of the Eiffel Tower).
- B' to name the place where R appears to be as seen from B (right side of the Eiffel Tower).

Now we'll draw the parallax picture again, but using the new notation and without the art. By studying Image 2, we will understand the parallax geometry and be able to use it as an astronomy tool. If the image is not clear to you, picture in the place of each letter the object it represents, you'll see it's no different from the picture you saw before.

It's clear in the Image 2, that the ABR triangle and the A'B'R triangle are proportional to each other. That is because, they share two edges and the third ones are parallel to each other. Now, if that two triangles are proportional, that means the relation between their edges longitudes is the same for both. For example, the relation between the left triangle's long edge $\overline{\mathrm{AR}}$ and it's short edge $\overline{\mathrm{AB}}$, which we could express as

$$\frac{\overline{\mathrm{AR}}}{\overline{\mathrm{AB}}}$$

must be the same relation as the one between the right triangle's long edge $\overline{\mathrm{A'R}}$ and it's short edge $\overline{\mathrm{A'B'}}$, which we could express as

$$\frac{\overline{\mathrm{A'R}}}{\overline{\mathrm{A'B'}}}$$

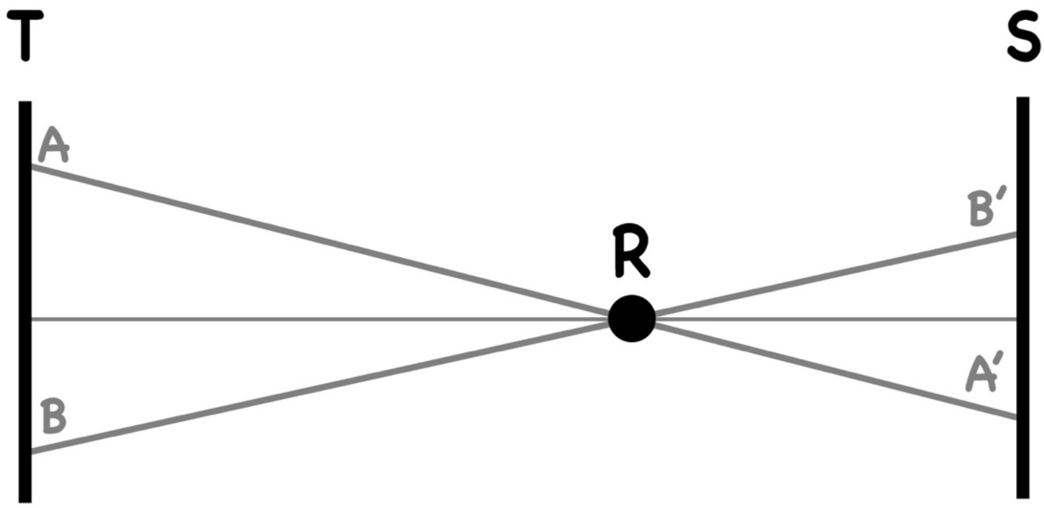

Image 2: Parallax Effect Geometry

So, in other words, we know that

$$\frac{\overline{AR}}{\overline{AB}} = \frac{\overline{A'R}}{\overline{A'B'}}$$

We will call this equation (eq. I).

Another little example coming next. Imagine you are in London, and you measure the distance to the Colosseum in Rome. Then, still from London, you measure the distance to the Trevi Fountain (that is in Rome too). Because the distance between London and Rome is huge, there is no point on making a difference between the distance to the Colosseum and the distance to the Trevi Fountain. You can just say that the distance to both is pretty much the same as the distance to Rome, and you'll be accurate enough.

Now, let's say R, S and T are apart from each other, actually, imagine they are so far away, that we can say that $\overline{AR}$ and $\overline{BR}$ are pretty much the same as $\overline{TR}$ (measured perpendicularly). So:

$$\overline{AR} = \overline{BR} = \overline{TR}$$

and in the same way

$$\overline{A'R} = \overline{B'R} = \overline{SR}$$

If this is true, we can change (eq. I) for

$$\frac{\overline{TR}}{\overline{AB}} = \frac{\overline{SR}}{\overline{A'B'}}$$

This means, that because the triangles in the picture are proportional to each other, the distance between A and B, $\overline{AB}$; and the distance between A' and B', $\overline{A'B'}$; are related to $\overline{TR}$ and $\overline{SR}$ by an equation that can also be written as

$$\frac{\overline{A'B'}}{\overline{SR}} = \frac{\overline{AB}}{\overline{TR}}$$

Usually this equation is enough to solve most of parallax situations, but the reader might keep reading if he wants to learn more relations from parallax geometry that will come handy in more complex situations. If not, feel free to skip the following, and read "Parallax in Astronomy" to finally accomplish the purpose of this booklet.

Parallax in astronomy

Now that you know fair enough about parallax geometry, you may be wondering how that is useful.

In the previous section, we found an equation that relates the distance between the exact location of two observers, $\overline{AB}$; the distance between where an object appears to be from those two places, $\overline{A'B'}$; the distance from that object two the observers, $\overline{TR}$; and the distance from that object to the background $\overline{SR}$. We can use this relation to measure distances in astronomy. We only have to observe an object from two different places, then, measuring the distance between those two places, measuring the distance between the apparent position of the object as seen from those two places, and using the relation we found, we can calculate the distance to the object, or the distance between the object and the background.

Let's see a few examples:

Look at Image 3, if we observe a near asteroid from two different places in Earth, we can use a distant-stars-background to measure the difference in the apparent position of the asteroid. Then, measuring the distance between the two observatories we can use our equation to find out how far the asteroid is.

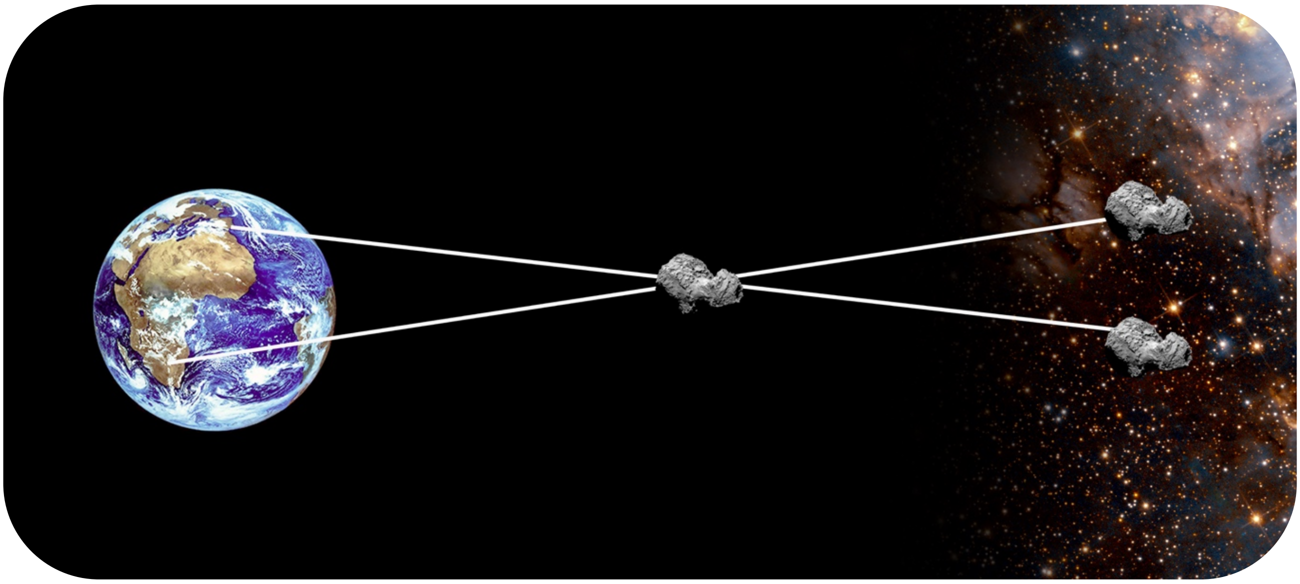

Image 3: Distance to an Asteroid

Let's see an other example in Image 4. If we observe the Sun from two different places during a transit. We can use the solar disk as background to measure the difference in the apparent position of the planet. Then, measuring the distance between the two observatories we can use our equation to find out how far the planet is from the Sun.

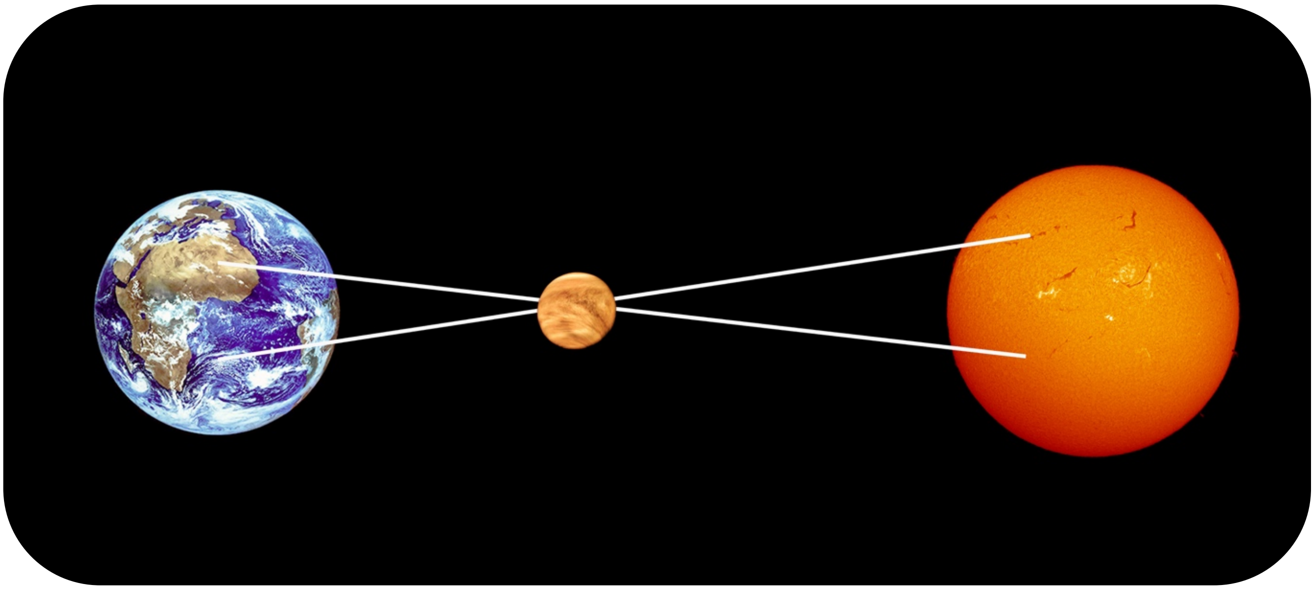

Image 4: Planet-Sun distance

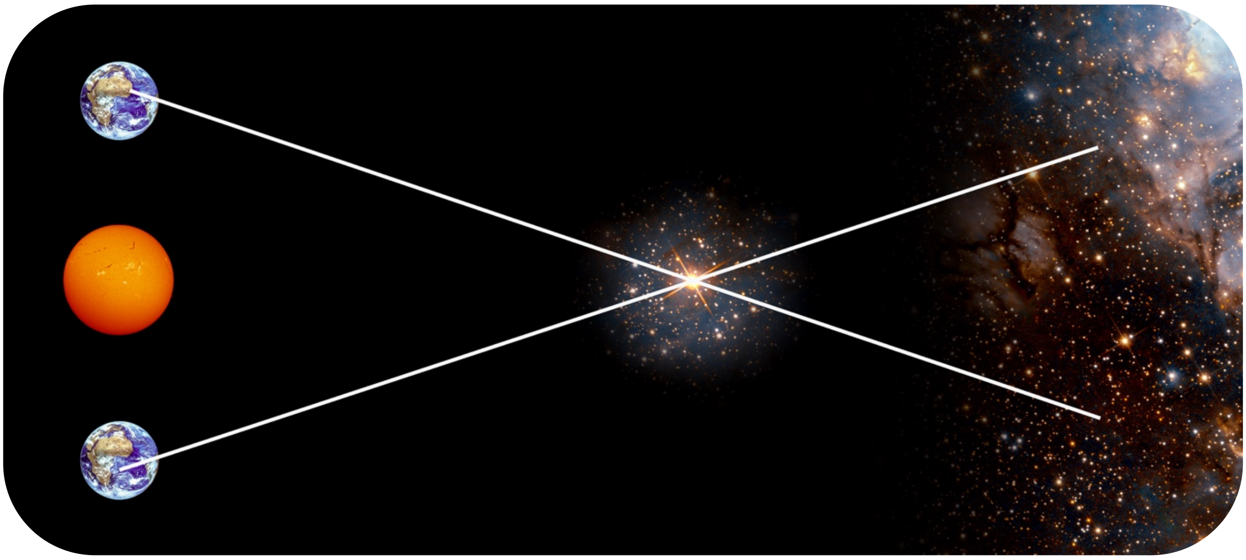

Image 5: Distance to a Star

In Image 5, if we observe a near star in the two equinox (same observatory, but Earth at two different positions). We can use a distant-stars-background to measure the difference in the apparent position of the star. Then, knowing the Earth-Sun distance we can use our equation to find out how far the star is