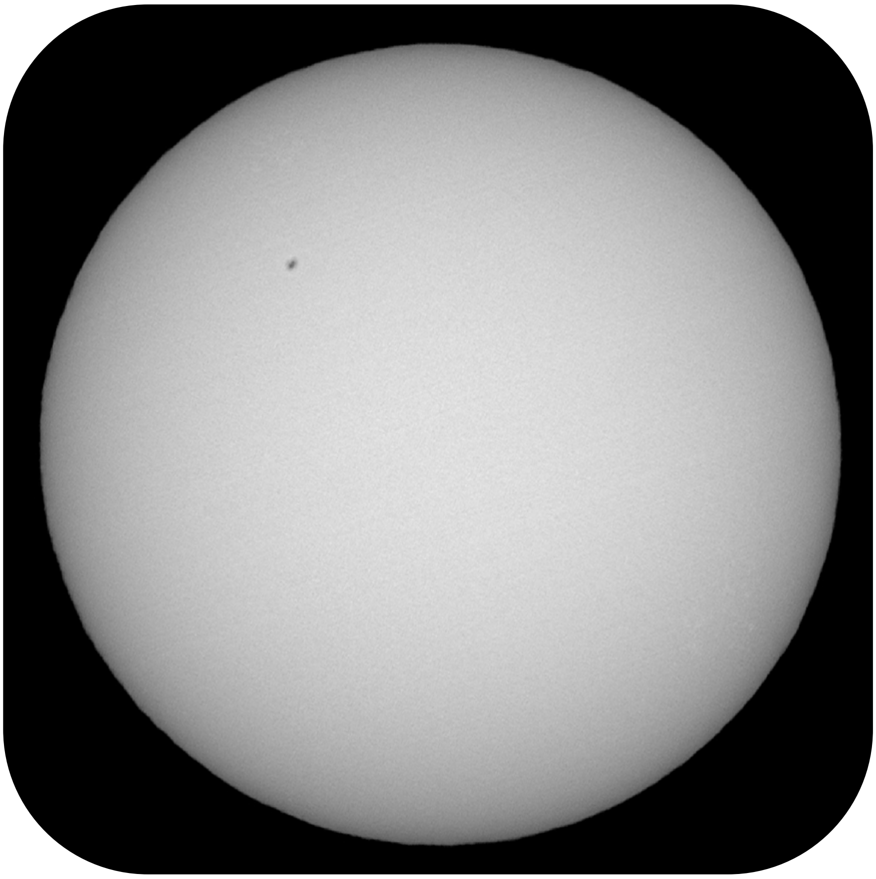
Heliographic Coordinates

CESAR's Booklet



It's the purpose of this booklet to obtain the coordinates of a sunspot seen in a Sun-picture.

Make sure to read slowly and comprehend every single sentence before further reading.

A sunspot seen from Earth

Look carefully at Image 1.

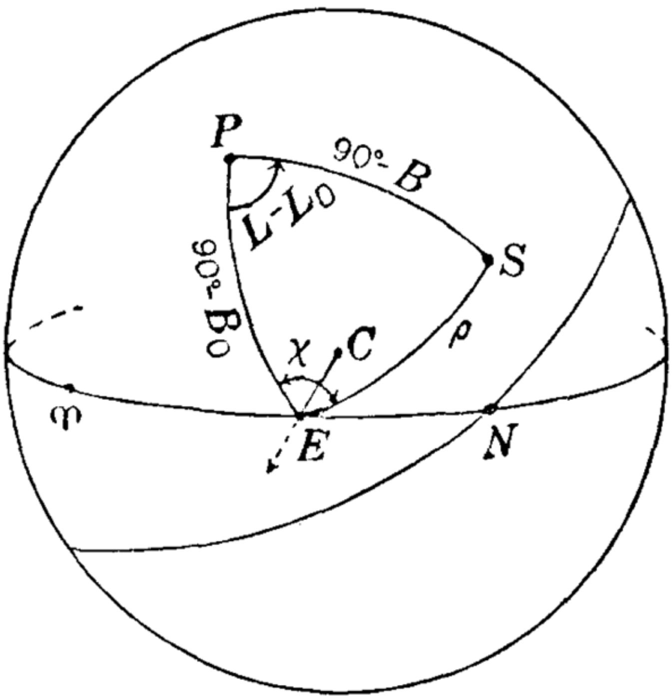


Image 1: Spherical Triangle in the Sun

Paint it yellow and it will look pretty much like the Sun. In the picture, S marks the position of a sunspot, which coordinates, B and L, we want to measure. E is just the centre of a Sun-picture taken from Earth, which is the same as saying that E is the perpendicular projection of Earth in the Sun. P is the North-Pole, that **is not** located at 90° from E because the Sun's rotation axis oscillates over time, so the North is not always on the top as seen from Earth. Still, the images taken by CESO (CESAR ESAC Solar Observatory) are properly rotated to compensate the obliquity of Earth's and Sun's rotation axis, so the point P is strictly above the centre of the image E, even if its not always at the same distance of the top (it may even be on the "back" of the image).

It happens to be that the angle between P and the top of the image taken from Earth (centred in C), can be obtained for each day from databases. If we know that angle, that we'll call B_0, then the angle between P and E is 90°- B_0. We will call the latitude of the sunspot S, B, so that the angle between P and S (centred in C) is 90°-B. The PES angle is going to be named θ. On the other hand, the EPS angle will be L-L_o, where L is the longitude of the sunspot S, and L_0 is the difference in longitude between E and the prime meridian. Finally the angle between E and S (centred in C) is ρ, that can be obtained if we have a Sun-image taken from Earth. We will use some other images to illustrate that:

Looking for ρ

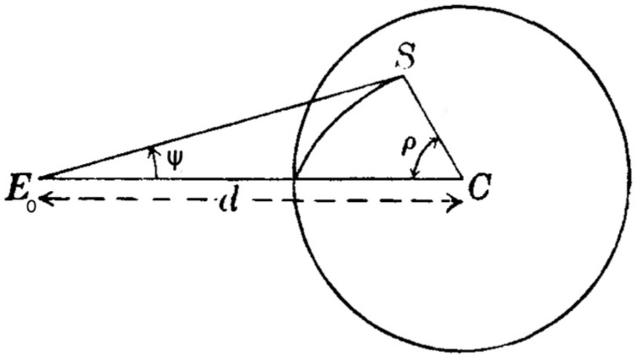


Image 2: Earth-Sun-Sunspot system

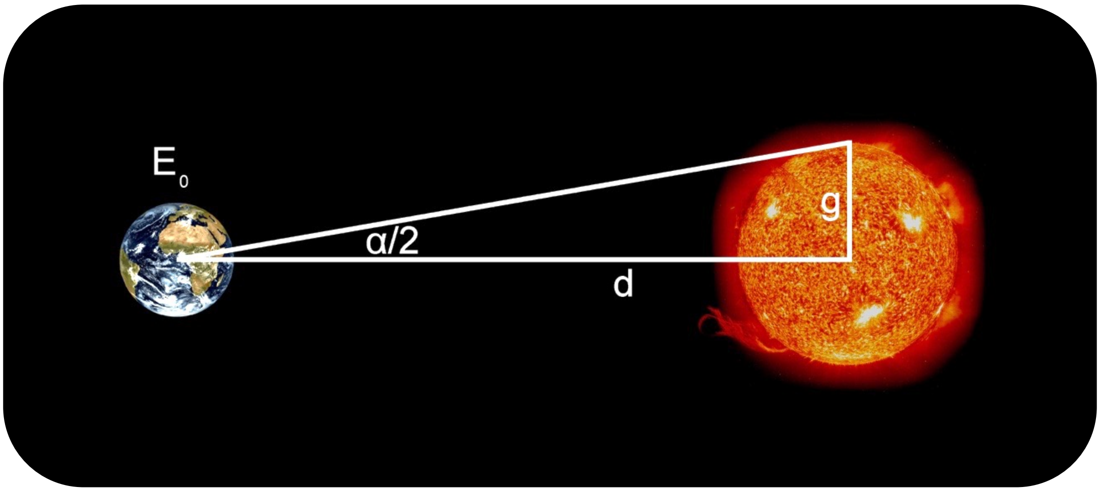


Image 3: Earth-Sun system

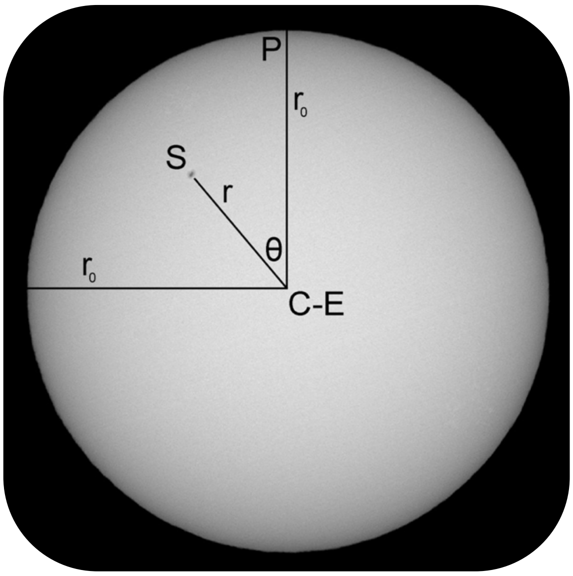


Image 4: Sun picture from Earth

The sphere in Image 2, is still supposed to be the Sun, but now we see the Earth E_0 in the 3D representation, and not as a projection in Sun's surface. ρ, which is the angle whose value we seek, is still the angle between E and S. In Image 2, the angle (centred in E_0) between the centre C and the sunspot S, ψ, is related to the distance between those two points, r, as measured in an image taken from Earth (Image 4), in the same way as in Image 3, the angle between the centre and the edge of Sun's disk $\alpha/2$ is related to the distance between them, r_0, measured in the Image 4. So we are confident to aver that

$$\frac{\psi}{\alpha/2} = \frac{r}{r_0}$$

Let's look closer at the E_0SC triangle now (Image 5) (note that g is the sun radius).

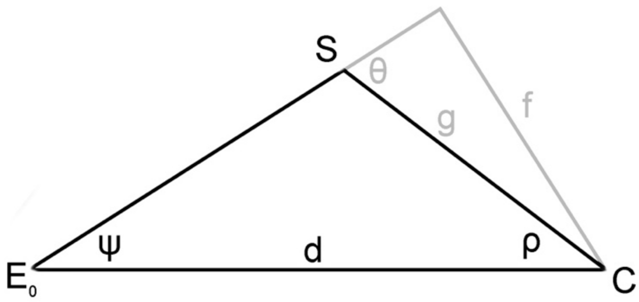


Image 5: Closer look of Image 2 (angles are distorted)

It's straightforward that

$$180° - \widehat{ESC} = \theta$$

and that

$$\widehat{ESC} = 180° - \psi - \rho$$

so

$$\sin\left(180° - \widehat{ESC}\right) = \frac{f}{g}$$

and also

$$\sin(\psi) = \frac{f}{d}$$

Merging both by equalling f to f, gives

$$\sin\left(180^\circ - \widehat{ESC}\right) \cdot g = \sin(\psi) \cdot d$$

which is the same as

$$\sin\left(\psi + \rho\right) \cdot g = \sin(\psi) \cdot d$$

and assuming that ψ is small

$$\sin\left(\psi + \rho\right) = \frac{\psi \cdot d}{g}$$

Besides, we know from Image 3 that

$$\sin\left(\alpha/2\right) = \frac{g}{d}$$

and assuming that $\alpha/2$ is small too

$$\frac{d}{g} = \frac{2}{\alpha}$$

Replacing that in the previous, gives

$$\sin\left(\psi + \rho\right) = \frac{\psi}{\alpha/2}$$

which can be transformed to get

$$\rho = \arcsin\left(\frac{\psi}{\alpha/2}\right) - \psi$$

that allows us to calculate ρ.

Cosine-Formula

Now that we know ρ (and the others angles defined at the very beginning), we can get back to the first picture and apply the cosine-formula there. Refer to the "Spherical Trigonometry" CESAR Booklet for further information about this formula. Once applied to the P, E and S points, one of the three possible cosine-formulas looks like this:

$$\cos\left(90°-B\right)=\cos(\rho)\cdot\cos\left(90°-B_0\right)+\sin(\rho)\cdot\sin\left(90°-B_0\right)\cdot\cos(\theta)$$

But we will simplify it and express it as

$$\sin(B)=\sin(B_0)\cdot\cos(\rho)+\cos(B_0)\cdot\sin(\rho)\cdot\cos(\theta)$$

So, using this equation, we can get the latitude of the sunspot S, B, if we previously know:

- B_0, which (as was said before) can be obtained from a database
- θ, which can be measured in the photography
- ρ, which can be calculated with the formula we developed if we know:
    - α, that changes over time, but can be found in a data base too
    - ψ, that is determined by the first relation in the "looking for ρ" section if we know:
        - α, that changes over time, but can be found in a data base too
        - r, which can be measured in the photography
        - r_0, which can be measured in the photography

You are free to look at the following databases to look for the needed values, although in the CESAR Web-Tools, the CESAR database is implemented when needed.

vo.imcce.fr/webservices/miriade/

http://bass2000.obspm.fr/home.php

About the quantities that can be measured in the photography:

r_0, can be measured in pixels, from the centre to the edge of the sun.

Then, if you measure the Cartesian coordinates (starting in the centre of the image) of the sunspot, x and y, you can get θ doing

$$\theta=\arctan\left(x/y\right)$$

and also you can get r using Pythagoras theorem.

At this point you should be able to obtain sunspot's latitude, B.

Sine-Formula

We have used the cosine-formula to calculate the latitude. It sounds fair to use the sine-formula to obtain the longitude. One of the three possible sine-formulas for the PES points looks like this:

$$\frac{\sin(L-L_0)}{\sin(\rho)} = \frac{\sin(\theta)}{\sin(90°-B)}$$

But we will simplify it and express it as

$$\sin(L-L_0) = \sin(\rho)\cdot\sin(\theta)\cdot\cos(B)$$

and the solve it for L:

$$L = \arcsin\big(\sin(\rho)\cdot\sin(\theta)\cdot\cos(B)\big) + L_0$$

Everything in this formula but L_0 should be familiar by now, and L_0 can be also obtained from databases.

And this is it! We now now B and L, the heliographic coordinates of a sunspot!